WDE
Opportunity Through Education

2021 WYOMING MATH

PERFORMANCE STANDARDS WITH 2018 CONTENT STANDARDS

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Content Standards Effective - July 12, 2018
TO BE FULLY IMPLEMENTED IN DISTRICTS BY THE BEGINNING OF SCHOOL YEAR 2021-22

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MATH WYOMING 2018 CONTENT AND 2021 PERFORMANCE STANDARDS

ACKNOWLEDGMENT: The Wyoming State Board of Education would like to thank the Wyoming Department of Education, as well as educators, parents and community members, business and industry representatives, community college representatives, and the University of Wyoming representatives for their work on the development of these Math Content Standards (CS), Performance Standards (PS), and Performance Level Descriptors (PLDs).

INTRODUCTION: The Wyoming Math Content and Performance Standards (WYCPS) were last reviewed and approved in 2012 in accordance with Wyoming State Statute W.S. 21-2-304(c). The 2018 Wyoming Math Content Standards were developed collaboratively through the contributions of Math Standard Review Committee (MSRC) members from across the state. In 2020, a new committee was convened with members from the original MSRC to identify the Performance Standards. For both reviews, the committee's work was informed and guided by initial public input through community forums, as well as input solicited from stakeholder groups.

RATIONALE: Mathematics is the language that defines the blueprint of the universe. Mathematics is woven into all parts of our lives and is more than a list of skills to be mastered. The essence of mathematics is the ability to employ critical thinking and reasoning to solve problems. To be successful in mathematics, one must see mathematics as sensible, useful, and worthwhile. The 2018 Wyoming Mathematics Content and Performance Standards address two kinds of knowledge: mathematical content and mathematical practice.

WHY DO WE HAVE STANDARDS FOR MATHEMATICS?

Uniform and consistent mathematical education is necessary as it ensures that all students in Wyoming are prepared for success in and out of the classroom. Therefore, the 2018 Wyoming Mathematics Content and Performance Standards:

Provide students, parents, and educators focus and coherence through application including understanding of mathematical concepts and processes.

- Align K-12 with clearly defined goals and outcomes for learning.
- Emphasize conceptual understanding.
- Encourage multiple models, representations, and strategies.
- Use technology to optimize mathematical understanding.

Develop students' mathematical thinking.

- Develop reasoning, solving, representing, proving, communicating, and connecting across contexts and applications.
- Recognize and identify mathematics in the world around us.
- Engage students in making sense, building conceptual understanding, developing procedural fluency, and employing adaptive reasoning.
- Build constructive attitudes to see mathematics as sensible, useful and worthwhile, and to increase confidence in one's own ability to do mathematics.

2021 Math Wyoming Content & Performance Standards

ORGANIZATION OF THE STANDARDS: (with terminology)

Standard Code: Grade. Math Domain. Cluster Code. Standard #
Key: K.G.I.4 = Grade **K**. Domain Geometry **(G).** Cluster **I**. Standard **4**

DOMAINS are the core concepts to be studied in math. The Math Standards usually consist of 5-6 domains in each grade level. The math domains are listed below.

- Kindergarten – Counting & Cardinality (CC)
- K-5 – Operations & Algebraic Thinking (OA)
- K-5 – Number & Operations in Base Ten (NBT)
- K-5 – Measurement & Data (MD)
- K-HS – Geometry (G)
- 3-5 – Number & Operations – Fractions (NF)
- 6-7 – Ratios & Proportional Relationships (RP)
- 8-12 – Functions (F)
- 6-8 – Expressions & Equations (EE)
- 6-8 – The Number System (NS)
- 6-12 – Statistics & Probability (SP)
- 9-12 – Number & Quantity (N)
- 9-12 –Algebra (A)

CONTENT STANDARDS define the content and skills students are expected to know and be able to do by the end-of-the-grade level or grade band. They are built foundationally and then in learning progressions. They do not dictate what methodology or instructional materials should be used, nor how the material is delivered. In this standards document, you will find these are broken out into individual grades for Kindergarten through 8th grade and then banded by domains for high school (9-12).

STANDARDS FOR MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ADVANCED STANDARDS (+)

The high school standards specify the mathematics that all students should study to be college and career ready. Each standard **without** a **(+)** symbol should be in the common mathematics curriculum for all students. Advanced mathematics standards, those designated **with** a **(+)** sign, are integrated into the higher level math courses after Algebra II. These standards encourage student experiences in higher level mathematical thinking and/or STEM pathways.
(Adapted from CCSS https://edu.wyoming.gov/downloads/standards/final-2012-math-standards.pdf)

PERFORMANCE LEVEL DESCRIPTORS (PLDs) describe the performance expectations of students for each of the four (4) performance level categories: Advanced, Proficient, Basic, and Below Basic. These are a description of what students within each performance level are expected to know and be able to do.

PERFORMANCE STANDARDS (PS) are the standards all students are expected to learn and be assessed on through the district assessment system by the end-of-the grade band. They specify the degree of understanding or demonstration of the knowledge and/or skill for a particular content standard. As such, they employ clear action verbs and describe “how good is good enough.”

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For those designated as PS, the content standard is marked with the code in blue highlight and an asterisk (*) and the Proficient PLD Statement is the PS expectation and is highlighted in blue.

Districts are expected to give students multiple opportunities to demonstrate proficiency on the Performance Standards through the District Assessment System (DAS) and provide appropriate supports for student success. **Teachers should provide extra focus, targeted supports, and offer multiple opportunities to demonstrate** student understanding (mastery). In the secondary level, only students electing to take a course aligned to these standards need to be assessed in the DAS.
A snapshot of the Math Performance Standards can be found on page 8.

MATHEMATICAL LITERACY

"Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged, and reflective citizens." https://www.achieve.org/files/StrongStandards.pdf

WHY DO WE HAVE THE STANDARDS FOR MATHEMATICAL PRACTICE?

Procedural knowledge alone will not prepare our 21st Century students to be globally competitive. Mathematical thinkers also visualize problems and recognize that multiple strategies may lead to a single solution**.** They realize mathematics is applicable outside of the classroom and are confident in their ability to apply mathematical concepts to all aspects of life. The Standards for Mathematical Practice cultivate mathematically literate and informed citizens. Using mathematics as a means of synthesizing complex concepts and making informed decisions is paramount to college and career success**.** The Standards for Mathematical Practice develop skills that serve students beyond the math classroom.
http://www.corestandards.org/Math/Practice/

COMPUTATIONAL THINKING

Computational thinking is necessary and meaningful in mathematics. Computational thinking has developed into competencies in problem solving, critical thinking, productivity, and creativity. Over time, engaging in computational thought builds a student's capacity to persevere, work efficiently, gain confidence, tolerate ambiguity, generalize concepts, and communicate effectively. In order to adapt to global advancements in technology, students will need to use their computational thinking skills to formulate, articulate, and discuss solutions in a meaningful manner.

MODELING

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

The basic modeling cycle involves: (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the

situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them.
http://www.corestandards.org/Math/Content/HSM/

Mathematics | Standards for Mathematical Practice

"The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). "
Source: http://www.corestandards.org/Math/Practice/

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs, or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning

of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

2021 Math Wyoming Content & Performance Standards

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HOW TO READ THIS DOCUMENT

The WYCPS have 3 main sections:

1) **Domain (**in black **bold)** with the **domain-specific learning expectations in blue, aka cluster headings**.
2) **Standard Code** and **Content Standard (CS) (**in black **bold)**.
3) **Performance Standard (PS)** – For the targeted subset of the Proficient PLD statements identified as the PS, the CS code are denoted with an asterisk (*) and highlighted in blue and the Proficient PLD (aka the PS) is highlighted in a lighter blue.

Kindergarten Math Content & Performance Standards

1

COUNTING AND CARDINALITY

Know number names and the count sequence.

***K.CC.A.1**

2

K.CC.A.1A Count to 100 by ones and by tens.

K.CC.A.1B Count backwards by ones from 20.

The Proficient student is able to:

3

A1. Count to 100 by ones, starting at one.
A2. Count to 100 by multiples of ten, starting at ten.
B. Count backwards by ones from 20.

K.CC.A.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with a 0 representing a count of no objects).

Count to tell the number of objects.

***K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality.**

K.CC.B.4A Use one-to-one correspondence when counting objects.
K.CC.B.4B Understand that the last number name said, tells the number of objects counted regardless of their arrangement.
K.CC.B.4C Understand that each successive number name refers to a quantity that is one more, and each previous number name refers to a quantity that is one less.

The Proficient student is able to count and tell the number of objects in a range from 10 to 39.

A. Use one-to-one correspondence when counting objects.
B. Understand that the last number name said, tells the number of objects counted regardless of their arrangement.
C. Understand that each successive number name refers to a quantity that is one more, and each previous number name refers to a quantity that is one less.

Kindergarten Math Content & Performance Standards

GRADE K MATH PRACTICES

MP1 Make sense of problems and persevere in solving them.

K.MP.1 In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students can explain the meaning of a problem and look for ways to solve it. Students check their thinking by using concrete objects or pictures to help them conceptualize and solve problems. Students are also working on increasing stamina as they work on problems.

MP2 Reason abstractly and quantitatively.

K.MP.2 Students begin to recognize what a number is and that it also represents a specific quantity. Then, they connect the quantity to written symbols. Students make meaning of word problems and use manipulatives to express and solve their thinking. Students are also working on increasing stamina as they work on problems.

MP3 Construct viable arguments and critique the reasoning of others.

K.MP.3 Students construct arguments using concrete illustrations, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions such as, “How did you get that? and Why is that true?” They explain their thinking to others and respond to others’ thinking by making connections. Students are also working on increasing stamina as they work on problems.

MP4 Model with mathematics.

K.MP.4 Students experiment with representing problem situations in multiple ways including using objects, acting out, drawing pictures, numbers, words (mathematical language), making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

MP5 Use appropriate tools strategically.

K.MP.5 Students begin to explore the different available tools when thinking about the concepts of numbers. They begin to learn which tools help strengthen their understanding of concepts. For instance, kindergarteners may decide that it might be advantageous to use linking cubes to represent two quantities and then compare the two representations side-by-side.

MP6 Attend to precision.

K.MP.6 As kindergarteners begin to develop their mathematical communication skills, they try to use clear and precise mathematical vocabulary in their discussions with others and in their own reasoning. Students learn to attend to the shapes of numbers, quickly recognize quantities (subitizing), and simple drawings to show their work.

MP7 Look for and make use of structure.

K.MP.7 Students begin to notice a number pattern or structure. For instance, students recognize the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated, and the pattern of numbers 0-9 repeat in the following numbers of 20, 30, etc. They also recognize that $3 + 2 = 5$ and $2 + 3 = 5$.

MP8 Look for and express regularity in repeated reasoning.

K.MP.8 Students notice repetitive actions in counting and computation, etc. For example, they may notice that the next number in a counting sequence is one more. When counting by tens, the next number in the sequence is ten more (or one more group of ten). Students also notice that when adding two numbers, order of adding doesn’t affect the sum (Commutative Property).

2021 Math Wyoming Content & Performance Standards

COUNTING AND CARDINALITY

Know number names and the count sequence.

***K.CC.A.1**

K.CC.A.1A Count to 100 by ones and by tens.

K.CC.A.1B Count backwards by ones from 20.

The **Proficient** student is able to:

A1. Count to 100 by ones, starting at one.
A2. Count to 100 by multiples of ten, starting at ten.
B. Count backwards by ones from 20.

K.CC.A.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with a 0 representing a count of no objects).

Count to tell the number of objects.

***K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality.**

K.CC.B.4A Use one-to-one correspondence when counting objects.

K.CC.B.4B Understand that the last number name said, tells the number of objects counted regardless of their arrangement.

K.CC.B.4C Understand that each successive number name refers to a quantity that is one more, and each previous number name refers to a quantity that is one less.

The **Proficient** student is able to count and tell the number of objects in a range from 10 to 39.

A. Use one-to-one correspondence when counting objects.
B. Understand that the last number name said, tells the number of objects counted regardless of their arrangement.
C. Understand that each successive number name refers to a quantity that is one more, and each previous number name refers to a quantity that is one less.

K.CC.B.5

K.CC.B.5A When counting, answer the question "how many?" by counting up to 20 objects arranged in a line, a rectangular array, a circle, or as many as 10 objects in a scattered configuration.

K.CC.B.5B When counting, given a number from 1-20, count out that many objects.

Compare numbers.

K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects.)

K.CC.C.7 Compare two numbers between 1 and 10 presented as written numerals.

2021 Math Wyoming Content & Performance Standards

OPERATIONS AND ALGEBRAIC THINKING

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.D1 Model situations that involve representing addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

***K.OA.D.2** **Solve word problems using objects and drawings to find sums up to 10 and differences within 10.**

The **Proficient** student is able to solve word problems using objects and drawings to find sums up to 10 and differences within 10.

***K.OA.D.3** **Decompose numbers less than or equal to 10 in more than one way.**

The **Proficient** student is able to decompose numbers less than or equal to 10 in more than one way.

K.OA.D.4 For any number from 1 to 9, find the number that makes 10 when added to the given number.

K.OA.D.5 Fluently add and subtract within 5.

NUMBERS AND OPERATIONS IN BASE TEN

Work with numbers 11-19 to gain foundations for place value.

K.NBT.E.1

K.NBT.E.1A Describe, explore, and explain how the counting numbers 11 to 19 are composed of ten ones and more ones.

K.NBT.E.1B Describe, explore, and explain how the counting numbers 11 to 19 are decomposed into ten ones and more ones.

MEASUREMENT AND DATA

Describe and compare measurable attributes.

K.MD.F.1 Describe several measurable attributes of one or more objects.

K.MD.F.2 Make direct comparisons of the length, capacity, weight, and temperature of objects, and recognize which object is shorter/longer, taller, lighter/heavier, warmer/cooler, and which holds more/less.

Classify objects and count the number of objects in each category.

***K.MD.G.3** **Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)**

The **Proficient** student is able to classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)

K.MD.G.4 Identify U.S. coins by name (pennies, nickels, dimes, and quarters).

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GEOMETRY

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.H.1 Describe objects in the environment using the names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.

K.G.H.2 Correctly name shapes regardless of their orientations or overall size.

K.G.H.3 Identify shapes as two-dimensional or three-dimensional.

Analyze, compare, create, and compose shapes.

K.G.I.4 Analyze and compare two- and three-dimensional shapes, using informal language to describe their similarities, differences, and attributes.

K.G.I.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

K.G.I.6 Use simple shapes to compose squares, rectangles, and hexagons.

Grade 1 Math Content & Performance Standards

GRADE 1 MATH PRACTICES

MP1 Make sense of problems and persevere in solving them.

1.MP.1 In first grade, students realize that doing mathematics involve solving problems and discussing how they solved them. Students explain the meaning of a problem and look for ways to solve it. Students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by revisiting their work and asking themselves, "Does this make sense?" or, "Should I try another strategy? Students are also working on increasing stamina as they work on problems.

MP2 Reason abstractly and quantitatively.

1.MP.2 Students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning means being able to explain through manipulatives or drawings what a problem means while attending to the meanings of the quantities. Students make meaning of a problem situation and translate into a number sentence.

MP3 Construct viable arguments and critique the reasoning of others.

1.MP.3 First graders construct arguments using concrete illustrations referents, such as objects, pictures, drawings, and actions. They also practice their mathematical communication skills as they participate in mathematical discussions involving questions like, "How did you get that?" Explain your thinking, "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask questions for clarity.

MP4 Model with mathematics.

1.MP.4 Students experiment with representing problem situations in multiple ways including using objects, acting out, drawing pictures, numbers, words (mathematical language), making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

MP5 Use appropriate tools strategically.

1.MP.5 Students begin to consider the different tools available when thinking about the concepts of number. They evaluate the available tools (including concrete manipulatives, drawings, estimation, and applications) when solving a mathematical problem and decide when certain tools might be helpful and give a reason for using the tool to solve the problem. For instance, first graders decide it might be best to use colored chips to model an addition problem.

MP6 Attend to precision.

1.MP.6 Students begin to develop their mathematical communication skills. They try to use clear and precise mathematical vocabulary in their discussions with others and in their own reasoning. Students learn to express their work with mathematical language and symbols.

MP7 Look for and make use of structure.

1.MP.7 First graders begin to discern a number pattern or structure. For instance, if students recognize$12 + 3 = 15$, then they also know $3 + 12 = 15$. (Commutative Property of addition.) To add $4 + 6 + 4$, the first two numbers can be added to make a ten, so $4 + 6 + 4 = 10 + 4 = 14$. Students continue to develop their understanding of patterns in our number system

MP8 Look for and express regularity in repeated reasoning.

1.MP.8 Students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract ten, including multiples of ten, then they notice the pattern and gain a better understanding of place value. Students also notice that when adding two numbers, order of adding doesn't affect the sum (Commutative Property). They also notice that

three numbers create a family when adding or subtracting ($2 + 3 = 5$ and $5 - 2 = 3$).

OPERATIONS AND ALGEBRAIC THINKING

Represent and solve problems involving addition and subtraction.

***1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, by using objects, drawings, or equations with a symbol for the unknown number to represent the problem.**

The **Proficient** student is able to use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, by using objects, drawings, or equations with a symbol for the unknown number to represent the problem.

1.OA.A.2 Solve word problems that call for the addition of three whole numbers whose sum is less than or equal to 20, by using objects, drawings, or equations.

Understand and apply properties of operations and the relationship between addition and subtraction.

1.OA.B.3 Apply Commutative and Associative Properties of addition as strategies to add and subtract.

1.OA.B.4 Understand subtraction as an unknown-addend problem.

Add and subtract within 20.

1.OA.C.5 Relate counting to addition and subtraction using strategies, such as, by counting on and back.

***1.OA.C.6 Add and subtract within 20, demonstrating fluency in addition and subtraction within 10. Use strategies such as counting on; making ten using the relationship between addition and subtraction.**

The **Proficient** student is able to add and subtract within 20, demonstrating fluency in addition and subtraction within 10. Use strategies such as counting on; making ten using the relationship between addition and subtraction.

Work with addition and subtraction equations.

1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.

1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

NUMBER AND OPERATIONS IN BASE TEN

Extend the counting sequence.

***1.NBT.E.1 Extend the number sequences to 120. In this range:**

- **1.NBT.E.1A** Count forward and backward, starting at any number less than 120.
- **1.NBT.E.1B** Read numerals.
- **1.NBT.E.1C** Write numerals.

1.NBT.E.1D Represent a number of objects with a written numeral.

The **Proficient** student is able to extend the number sequences to 120. In this range:

A. Count forward and backward, starting at any number less than 120.
B. Read numerals.
C. Write numerals.
D. Represent a number of objects with a written numeral.

Understand place value.

1.NBT.F.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

1.NBT.F.2A 10 can be thought of as a bundle of ten ones — called a "ten."
1.NBT.F.2B The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
1.NBT.F.2C The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

1.NBT.F.3 Compare pairs of two-digit numbers based on the values of the tens digit and the ones digits, recording the results of comparisons with the words "is greater than," "is equal to," "is less than," and with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

***1.NBT.G.4** **Add within 100, using concrete models or drawings and strategies based on place value:**

1.NBT.G.4A Including adding a two-digit number and a one-digit number.
1.NBT.G.4B Adding a two-digit number and a multiple of 10.
1.NBT.G.4C Understand that in adding two-digit numbers, adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
1.NBT.G.4D Relate the strategy to a written method and explain the reasoning used.

The **Proficient** student is able to add within 100, using concrete models or drawings and strategies based on place value:

A. Including adding a two-digit number and a one-digit number.
B. Adding a two-digit number and a multiple of 10.
C. Understand that in adding two-digit numbers, adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
D. Relate the strategy to a written method and explain the reasoning used.

1.NBT.G.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

1.NBT.G.6 Subtract multiples of 10 from an equal or larger multiple of 10 both in the range 10-90, using concrete models, drawings, and strategies based on place value.

MEASUREMENT AND DATA

Measure lengths indirectly and by iterating length units.

1.MD.H.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.

1.MD.H.2 Use nonstandard units to show the length of an object as the number of same size units of length with no gaps or overlaps.

2021 Math Wyoming Content & Performance Standards

Work with time and money.

***1.MD.I.3A Tell and write time in hours and half-hours using analog and digital clocks.**

The **Proficient** student is able to tell and write time in hours and half-hours using analog and digital clocks.

***1.MD.I.3B Identify U.S. coins by value (pennies, nickels, dimes, quarters).**

The **Proficient** student is able to identify U.S. coins by value (pennies, nickels, dimes, quarters).

Represent and interpret data.

1.MD.J.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

GEOMETRY

Reason with shapes and their attributes.

1.G.K.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); for a wide variety of shapes; build and draw shapes to possess defining attributes.

1.G.K.2 Use two-dimensional shapes (rectangles, squares, trapezoids, rhombuses, and triangles) or three-dimensional shapes (cubes, rectangular prisms, cones, and cylinders) to create a composite figure, and create new figures from the composite figure.

1.G.K.3 Partition circles and rectangles into two and four equal shares and:

- **1.G.K.3A** Describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of.
- **1.G.K.3B** Describe the whole as two of, or four of the shares.
- **1.G.K.3C** Recognize that decomposing into more equal shares creates smaller shares.

Grade 2 Math Content & Performance Standards

GRADE 2 MATH PRACTICES

MP1 Make sense of problems and persevere in solving them.

2.MP.1 In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They make conjectures about the solution and plan out a problem-solving approach. Students work on increasing stamina.

MP2 Reason abstractly and quantitatively.

2.MP.2 Students recognize that a number represents a specific quantity and connect the quantity to written symbols. Quantitative reasoning entails being able to explain through manipulatives or drawings what a problem means, while attending to the meanings of the quantities. Students make meaning of a problem situation and translate into a number sentence. Second graders begin to know and use different properties of operations and relate addition and subtraction.

MP3 Construct viable arguments and critique the reasoning of others.

2.MP.3 Second graders may construct arguments using concrete illustrations, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like, "How did you get that?" Explain your thinking, "Why is that true?" They not only explain their own thinking, but listen to others' explanations and compare strategies. They decide if the explanations make sense and ask appropriate questions for clarity.

MP4 Model with mathematics.

2.MP.4 Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

MP5 Use appropriate tools strategically.

2.MP.5 Students decide how and when to use the available tools appropriately and efficiently when solving a mathematical problem. Students reason whether or not a tool was helpful in solving the problem. For instance, second graders may decide to solve a problem by drawing a picture rather than writing an equation.

MP6 Attend to precision.

2.MP.6 Students begin to develop their mathematical communication skills, (orally and written). They use clear and precise mathematical language and symbols when explaining their own reasoning.

MP7 Look for and make use of structure.

2.MP.7 Second graders look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles, adding and subtracting numbers by place, and equal shares). Their understanding of the number system develops into 3- and 4- digit numbers.

MP8 Look for and express regularity in repeated reasoning.

2.MP.8 Second grade students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract, they look for shortcuts, such as tens are added to tens, ones are added to ones, and sometimes the ones make a new ten. They also notice when a whole is shared into equal groups, the size of the share gets smaller the more shares.

2021 Math Wyoming Content & Performance Standards

OPERATIONS AND ALGEBRAIC THINKING

Represent and solve problems involving addition and subtraction.

***2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, by using drawings and equations with a symbol for the unknown number to represent the problem.**

The **Proficient** student is able to use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, and taking apart, and comparing with unknowns in all positions, by using drawings and equations with a symbol for the unknown number to represent the problem.

Add and subtract within 20.

***2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know automatically all sums of two one-digit numbers based on strategies.**

The **Proficient** student is able to fluently add and subtract within 20 using mental strategies. By end of Grade 2, know automatically all sums of two one-digit numbers based on strategies.

Work with equal groups of objects to gain foundations for multiplication.

2.OA.C.3 Determine whether a group (up to 20) has an odd or even number of objects (i.e., by pairing objects or counting them by 2s).

2.OA.C.3A If the number of objects is even, then write an equation to express this as the sum of two equal addends.

2.OA.C.3B If the number of objects group is odd, then write an equation to express this as a sum of a near double (double plus 1).

2.OA.C.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

NUMBER AND OPERATIONS IN BASE TEN

Understand place value.

***2.NBT.D.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; and demonstrate that cases: a. 100 can be thought of as a bundle of ten tens — called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).**

2.NBT.D.1A 100 can be thought of as a bundle of ten tens — called a "hundred."

2.NBT.D.1B The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.D.1C Three-digit numbers can be decomposed in multiple ways (e.g., 524 can be decomposed as 5 hundreds, 2 tens and 4 ones or 4 hundreds, 12 tens, and 4 ones, etc.)

The **Proficient** student is able to understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; and demonstrate that:

A. 100 can be thought of as a bundle of ten tens — called a "hundred."

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B. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
C. Three-digit numbers can be decomposed in multiple ways (e.g., 524 can be decomposed as 5 hundreds, 2 tens and 4 ones or 4 hundreds, 12 tens, and 4 ones, etc.)

2.NBT.D.2 Skip-count by 10s and 100s within 1000 starting at any given number.

2.NBT.D.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.NBT.D.4 Compare pairs of three-digit numbers based on meanings of the hundreds, tens, and ones digits, using the words "is greater than," "is equal to," "is less than," and with the symbols >, =, and < to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

2.NBT.E.5 Add and subtract within 100 using strategies based on place value, properties of addition, and/or the relationship between addition and subtraction.

2.NBT.E.6 Add up to four two-digit numbers using strategies based on place value and/or properties of addition.
***2.NBT.E.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of addition, and/or the relationship between addition and subtraction:**
- **2.NBT.E.7A** Relate the strategy to a written method and explain the reasoning used.
- **2.NBT.E.7B** Understand that in adding or subtracting three-digit numbers, add or subtract hundreds and hundreds, tens and tens, ones and ones.
- **2.NBT.E.7C** Understand that sometimes it is necessary to compose or decompose tens or hundreds.

The **Proficient** student is able to add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of addition, and/or the relationship between addition and subtraction:
A. Relate the strategy to a written method and explain the reasoning used.
B. Understand that in adding or subtracting three-digit numbers, add or subtract hundreds and hundreds, tens and tens, ones and ones.
C. Understand that sometimes it is necessary to compose or decompose tens or hundreds.

2.NBT.E.8
- **2.NBT.E.8A Mentally, add 10 or 100 to a given number 100-900.**
- **2.NBT.E.8B Mentally, subtract 10 or 100 from a given number 100-900.**

2.NBT.E.9 Explain why addition and subtraction strategies work, using place value and the properties of addition. (Explanations may be supported by drawings, objects, or written form.)

MEASUREMENT AND DATA

Measure and estimate lengths in standard units.

2.MD.F.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.F.2 Measure the same object or distance using a standard unit of one length and then a standard unit of a different length. Explain how the two measurements relate to the size of the unit chosen.

2.MD.F.3 Estimate lengths using units of inches, feet, centimeters, and meters.

2.MD.F.4 Measure in standard length units to determine how much longer one object is than another.

Relate addition and subtraction to length.

2.MD.G.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units.

2.MD.G.6 Use a number line diagram with equally spaced points to:

- **2.MD.G.6A** Represent whole-number sums and differences within 100 on a number line diagram.
- **2.MD.G.6B** Locate the multiple of 10 before and after a given number within 100.

Work with time and money.

2.MD.H.7 Tell and write time from analog and digital clocks in five minute increments using a.m. and p.m.

***2.MD.H.8 Solve word problems up to $10 involving dollar bills, quarters, dimes, nickels, and pennies, using $ (dollars) and ¢ (cents) symbols appropriately.**

The **Proficient** student is able to solve word problems up to $10 involving dollar bills, quarters, dimes, nickels, and pennies, using $ (dollars) and ¢ (cents) symbols appropriately.

Represent and interpret data.

2.MD.I.9 Generate measurement data based on whole units and show data by making a line plot.

2.MD.I.10

- **2.MD.I.10A Use data to draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories.**
- **2.MD.I.10B Use data to solve simple put-together, take-apart, and compare problems using information presented in a bar graph.**

GEOMETRY

Reason with shapes and their attributes.

2.G.J.1 Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (Sizes are compared directly or visually, not compared by measuring.)

2.G.J.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

***2.G.J.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.**

- **2.G.J.3A** Describing the shares using the words halves, thirds, half of, a third of, etc.
- **2.G.J.3B** Describing the whole as two halves, three thirds, four fourths.
- **2.G.J.3C** Recognizing that equal shares of identical wholes need not have the same shape.

The **Proficient** student is able to partition circles and rectangles into two, three, or four equal shares by:

A. Describing the shares using the words halves, thirds, half of, a third of, etc.
B. Describing the whole as two halves, three thirds, four fourths.
C. Recognizing that equal shares of identical wholes need not have the same shape.

Grade 3 Math Content & Performance Standards

GRADE 3 MATH PRACTICES

MP1 Make sense of problems and persevere in solving them.

3.MP.1 In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third grade students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense? "They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

MP2 Reason abstractly and quantitatively.

3.MP.2 Students recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.

MP3 Construct viable arguments and critique the reasoning of others.

3.MP.3 Students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions such as, "How did you get that? and "Why is that true?" They explain their thinking to others and respond to others' thinking.

MP4 Model with mathematics.

3.MP.4 Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Students should evaluate their results in the context of the situation and reflect on whether the results make sense.

MP5 Use appropriate tools strategically.

3.MP.5 Students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.

MP6 Attend to precision.

3.MP.6 As students develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.

MP7 Look for and make use of structure.

3.MP.7 Students look closely to discover a pattern or structure. For example, students use properties of operations as strategies to multiply and divide (Commutative and Distributive Properties).

MP8 Look for and express regularity in repeated reasoning.

3.MP.8 Students notice repetitive actions in computation and look for shortcut methods. For example, students may use the Distributive Property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of $7 \cdot 8$, they might decompose 7 into 5 and 2 then multiply $5 \cdot 8$ and $2 \cdot 8$ to arrive at $40 + 16$ or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

2021 Math Wyoming Content & Performance Standards

OPERATIONS AND ALGEBRAIC THINKING

Represent and solve problems involving multiplication and division.

3.OA.A.1 Represent the concept of multiplication of whole numbers using models including, but not limited to, equal-sized groups ("groups of"), arrays, area models, repeated addition, and equal "jumps" on a number line.

3.OA.A.2 Represent the concept of division of whole numbers (resulting in whole number quotients) using models including, but not limited to, partitioning, repeated subtraction, sharing, and inverse of multiplication.

3.OA.A.3 Solve multiplication and division word problems within 100 using appropriate modeling strategies and equations.

3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is a missing factor, product, dividend, divisor, or quotient. (Students need not know formal terms.)

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.B.5 Apply properties of multiplication as strategies to multiply and divide. (Students need not use formal terms for these properties.)

3.OA.B.6 Understand division as an unknown-factor problem.

Multiply and divide within 100.

***3.OA.C.7** **Fluently multiply and divide with factors 1 - 10 using mental strategies. By end of Grade 3, know automatically all products of one-digit factors based on strategies.**

The **Proficient** student is able to fluently multiply and divide with factors 1 - 10 using mental strategies. By end of Grade 3, know automatically all products of one-digit factors based on strategies.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

***3.OA.D.8** **Solve two-step word problems (limited to the whole number system) using the four basic operations. Students should apply the Order of Operations when there are no parentheses to specify a particular order.**

3.OA.D.8A Represent these problems using equations with a symbol standing for the unknown quantity.

3.OA.D.8B Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

The **Proficient** student is able to solve two-step word problems (limited to the whole number system) using the four basic operations. Students should apply the Order of Operations when there are no parentheses to specify a particular order.

A. Represent these problems using equations with a symbol standing for the unknown quantity.

B. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

3.OA.D.9 Identify arithmetic patterns and explain the relationships using properties of operations.

2021 Math Wyoming Content & Performance Standards

NUMBER AND OPERATIONS IN BASE TEN

Use place value understanding and properties of operations to perform multi-digit arithmetic (a range of algorithms may be used).

3.NBT.E.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

***3.NBT.E.2** **Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of addition, and/or the relationship between addition and subtraction.**

The **Proficient** student is able to fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of addition, and/or the relationship between addition and subtraction.

3.NBT.E.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., $9 \cdot 80$, $5 \cdot 60$) using strategies based on place value and properties of multiplication.

NUMBER AND OPERATIONS - FRACTIONS

Develop understanding of fractions as numbers. (Limited to denominators 2, 3, 4, 6, and 8) *use horizontal fractions.

3.NF.F.1 Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

***3.NF.F.2** **Understand and represent fractions on a number line diagram.**

3.NF.F.2A Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

3.NF.F.2B Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

The **Proficient** student is able to understand and represent fractions on a number line diagram.

A. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

B. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

Assessment Boundary: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

***3.NF.F.3** **Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.**

3.NF.F.3A Understand two fractions as equivalent if they are the same size, or the same point on a number line.

3.NF.F.3B Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent.

3.NF.F.3C Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
3.NF.F.3D Compare two fractions with the same numerator or the same denominator, by reasoning about their size, Recognize that valid comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.

The **Proficient** student is able to explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

A. Understand two fractions as equivalent if they are the same size, or the same point on a number line.
B. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent.
C. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
D. Compare two fractions with the same numerator or the same denominator, by reasoning about their size. Recognize that valid comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.

Assessment Boundary: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

MEASUREMENT AND DATA

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.G.1 Use analog clocks to tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes.

3.MD.G.2 Measure and estimate liquid volumes and masses of objects using grams (g), kilograms (kg), and liters (L). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. (Excludes multiplicative comparison problems involving notions of "times as much.")

Represent and interpret data.

3.MD.H.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled graphs.

3.MD.H.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Use the data to create a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.I.5 Understand area as an attribute of plane figures and understand concepts of area measurement, such as square units without gaps or overlaps.

3.MD.I.6 Measure areas by counting unit squares (square cm, square m, square in., square ft., and improvised units).

***3.MD.I.7** **Relate area to the operations of multiplication and addition.**

3.MD.I.7A Find the area of a rectangle with whole-number side lengths (dimensions) by multiplying them. Show that this area is the same as when counting unit squares.
3.MD.I.7B Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
3.MD.I.7C Use area models to represent the Distributive Property in mathematical reasoning. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \cdot b$ and $a \cdot c$.

2021 Math Wyoming Content & Performance Standards

The **Proficient** student is able to relate area to the operations of multiplication and addition.

A. Find the area of a rectangle with whole-number side lengths (dimensions) by multiplying them. Show that this area is the same as when counting unit squares.

B. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

C. Use area models to represent the Distributive Property in mathematical reasoning. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \cdot b$ and $a \cdot c$.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.J.8 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different area or with the same area and different perimeter.

GEOMETRY

Reason with shapes and their attributes.

3.G.K.1 Use attributes of quadrilaterals to classify rhombuses, rectangles, and squares. Understand that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

3.G.K.2 Partition rectangles, regular polygons, and circles into parts with equal areas. Express the area of each part as a unit fraction of the whole.

Grade 4 Math Content & Performance Standards

GRADE 4 MATH PRACTICES

MP1 Make sense of problems and persevere in solving them.

4.MP.1 In grade four, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third grade students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense? “They listen to the strategies of others and will try different approaches. They will often use another method to check their answers.

MP2 Reason abstractly and quantitatively.

4.MP.2 Students recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.

MP3 Construct viable arguments and critique the reasoning of others.

4.MP.3 Students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.

MP4 Model with mathematics.

4.MP.4 Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect different representations and explain the connections. They should be able to use all of these representations as needed. Students should evaluate their results in the context of the situation and reflect on whether the results make sense.

MP5 Use appropriate tools strategically.

4.MP.5 Students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals, they may use protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.

MP6 Attend to precision.

4.MP.6 As students develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and stating the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.

MP7 Look for and make use of structure.

4.MP.7 Students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule

MP8 Look for and express regularity in repeated reasoning.

4.MP.8 Students notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and

generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

OPERATIONS AND ALGEBRAIC THINKING

Use the four operations with whole numbers to solve problems.

4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, by using strategies including, but not limited to, drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

***4.OA.A.3 Solve multi-step word problems posed with whole numbers, including problems in which remainders must be interpreted.**

4.OA.A.3A Represent these problems using equations with a letter standing for the unknown quantity.

4.OA.A.3B Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

The **Proficient** student is able to solve multi-step word problems posed with whole numbers, including problems in which remainders must be interpreted.

A. Represent these problems using equations with a letter standing for the unknown quantity.

B. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Develop understanding of factors and multiples.

4.OA.B.4 Demonstrate an understanding of factors and multiples.

4.OA.B.4A Find all factor pairs for a whole number in the range 1-100.

4.OA.B.4B Recognize that a whole number is a multiple of each of its factors.

4.OA.B.4C Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number.

4.OA.B.4D Determine whether a given whole number in the range 1-100 is prime or composite.

Generate and analyze patterns.

4.OA.C.5 Given a pattern, explain a rule that the pattern follows and extend the pattern. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

NUMBER AND OPERATIONS IN BASE TEN

Generalize place value understanding for multi-digit whole numbers (limited to numbers less than or equal to 1,000,000).

4.NBT.D.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

4.NBT.D.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols.

4.NBT.D.3 Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic (limited to whole numbers less than or equal to 1,000,000).

4.NBT.E.4 Add and subtract multi-digit whole numbers using place value strategies including the standard algorithm.

***4.NBT.E.5 Use strategies based on place value and the properties of multiplication.**

4.NBT.E.5A Multiply a whole number of up to four digits by a one-digit whole number.
4.NBT.E.5B Multiply a pair of two-digit numbers.
4.NBT.E.5C Use appropriate models to explain the calculation, such as by using equations, rectangular arrays, and/or area models.

The **Proficient** student is able to use strategies based on place value and the properties of multiplication to:

A. Multiply a whole number of up to four digits by a one-digit whole number.
B. Multiply a pair of two-digit numbers.
C. Use appropriate models to explain the calculation, such as by using equations, rectangular arrays, ratio tables, or area models.

***4.NBT.E.6 Use strategies based on place value, the properties of multiplication, and/or the relationship between multiplication and division to find quotients and remainders with up to four-digit dividends and one-digit divisors. Use appropriate models to explain the calculation, such as by using equations, rectangular arrays, and/or area models.**

The **Proficient** student is able to use strategies based on place value, the properties of multiplication, and/or the relationship between multiplication and division to find quotients and remainders with up to four-digit dividends and one-digit divisors. Use appropriate models to explain the calculation, such as by using equations, rectangular arrays, ratio tables, or area models.

NUMBER AND OPERATIONS - FRACTIONS

Extend understanding of fraction equivalence and ordering (limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100).

4.NF.F.1 Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n \cdot a}{n \cdot b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

***4.NF.F.2 Compare two fractions with different numerators and different denominators by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$.**

4.NF.F.2A Recognize that comparisons are valid only when the two fractions refer to the same whole.
4.NF.F.2B Record the results of comparisons with symbols >, =, or <.
4.NF.F.2C Justify the conclusions by using a visual fraction model.

The **Proficient** student is able to compare two fractions with different numerators and different denominators by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$.

A. Recognize that comparisons are valid only when the two fractions refer to the same whole.
B. Record the results of comparisons with symbols >, =, or <.
C. Justify the conclusions by using a visual fraction model.

Assessment Boundary: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8, 10, 12, and 100.

2021 Math Wyoming Content & Performance Standards

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers (limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100).

***4.NF.G.3 Understand a fraction a/b with $a > 1$ as a sum of unit fractions (1/b).**

4.NF.G.3A Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

4.NF.G.3B Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions by using a visual fraction model.

4.NF.G.3C Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction, and/or by using properties of addition and the relationship between addition and subtraction.

4.NF.G.3D Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.

The **Proficient** student is able to understand a fraction a/b with $a > 1$ as a sum of unit fractions ($\frac{1}{b}$).

A. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
B. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions by using a visual fraction model.
C. Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction, and/or by using properties of addition and the relationship between addition and subtraction.
D. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.

4.NF.G.4 Apply and extend an understanding of multiplication by multiplying a whole number and a fraction.

4.NF.G.4A Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$.

4.NF.G.4B Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number.

4.NF.G.4C Solve real-world problems involving multiplication of a fraction by a whole number, using visual fraction models and equations to represent the problem.

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.H.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.

4.NF.H.6 Use decimal notation for fractions with denominators 10 or 100.

4.NF.H.7 Compare and order decimal numbers to hundredths and justify by using concrete and visual models. Record the results of comparisons with the words "is greater than," "is equal to," "is less than," and with the symbols $>$, $=$, and $<$.

MEASUREMENT AND DATA

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.I.1 Know relative sizes of measurement units within one system of units including, but not limited to, km, m, cm; kg, g; lb., oz.; l L, ml; hr., min, sec; ft., in., gal., qt. pt., c. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.

4.MD.I.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
Assessment Boundary: Use denominators of 2, 4, 8 and decimals up to hundredths.

***4.MD.I.3 Apply the area and perimeter formulas for rectangles in real-world and mathematical problems.**
The **Proficient** student is able to apply the area and perimeter formulas for rectangles in real-world and mathematical problems.

Represent and interpret data.

4.MD.J.4 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots.

Geometric measurement: understand concepts of angle and measure angles.

4.MD.K.5
4.MD.K.5A Regarding angles, recognize angles as geometric shapes that are formed wherever two rays share a common endpoint.
4.MD.K.5B Regarding angles, understand concepts of angle measurement. An angle is measured with reference to a circle with its center at the common endpoint of the rays.

4.MD.K.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.K.7 Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.

GEOMETRY

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.L.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.L.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

4.G.L.3 Identify line-symmetric figures. Recognize and draw lines of symmetry for two-dimensional figures.

Grade 5 Math Content & Performance Standards

GRADE 5 MATH PRACTICES

MP1 Make sense of problems and persevere in solving them.

5.MP.1 In grade five, students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"

MP2 Reason abstractly and quantitatively.

5.MP.2 Students recognize that a number represents a specific quantity. They connect quantities to written symbols and create logical representation of the problem at hand, while considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.

MP3 Construct viable arguments and critique the reasoning of others.

5.MP.3 Students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.

MP4 Model with mathematics.

5.MP.4 Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, to create equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

MP5 Use appropriate tools strategically.

5.MP.5 Fifth graders consider the available tools, including estimation, when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems, or to make predictions from real-world data.

MP6 Attend to precision.

5.MP.6 Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism, they record their answers in cubic units.

MP7 Look for and make use of structure.

5.MP.7 Students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.MP8 Look for and express regularity in repeated reasoning.

2021 Math Wyoming Content & Performance Standards

MP8 Look for and express regularity in repeated reasoning.

5.MP.8 Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and to perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

OPERATIONS AND ALGEBRAIC THINKING

Write, interpret, and/or evaluate numerical expressions.

***5.OA.A.1** **Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.**

The **Proficient** student is able to use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.A.2 Write simple expressions requiring parentheses that record calculations with numbers, and interpret numerical expressions without evaluating them.

Analyze patterns and relationships.

5.OA.B.3 Generate two numerical patterns with each pattern having its own rule. Explain informally the relationship(s) between corresponding terms in the two patterns.

5.OA.B.3A Form ordered pairs consisting of corresponding terms from the two patterns.

5.OA.B.3B Graph the ordered pairs on a coordinate plane.

NUMBER AND OPERATIONS IN BASE TEN

Understand the place value system.

5.NBT.C.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

5.NBT.C.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.

5.NBT.C.3 Read, write, and compare decimals to thousandths.

5.NBT.C.3A Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.

5.NBT.C.3B Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols.

5.NBT.C.4 Use place value understanding to round decimals to any place to a given place.

Assessment Boundary: Limit place value to the thousandths.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.D.5 Multiply multi-digit whole numbers using place value strategies including the standard algorithm.

5.NBT.D.6 Find whole-number quotients with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of multiplication, and/or the relationship between multiplication and division, including the standard algorithm. Use appropriate models to Illustrate and explain the calculation, such as equations, rectangular arrays, and/or area models.

Assessment Boundary: The standard algorithm for division will not be assessed.

***5.NBT.D.7** **Add, subtract, multiply, and divide decimals to hundredths using concrete models or drawings, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; Relate the strategy to a written method and explain the reasoning used.**

The **Proficient** student is able to add, subtract, multiply, and divide decimals to hundredths using concrete models or drawings, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; Relate the strategy to a written method and explain the reasoning used.

NUMBER AND OPERATIONS - FRACTIONS

Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.E.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

***5.NF.E.2** **Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.**

The **Proficient** student is able to solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.F.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models or equations to represent the problem.

5.NF.F.4 Extend the concept of multiplication to multiply a fraction or whole number by a fraction.

5.NF.F.4A Recognize the relationship between multiplying fractions and finding the areas of rectangles with fractional side lengths.

5.NF.F.4B Interpret multiplication of a fraction by a whole number and a whole number by a fraction and compute the product.

5.NF.F.4C Interpret multiplication in which both factors are fractions less than one and compute the product.

5.NF.F.5 Justify the reasonableness of a product when multiplying with fractions.

5.NF.F.5A Estimate the size of the product based on the size of the two factors.

5.NF.F.5B Explain why multiplying a given number by a number greater than 1 (improper fractions, mixed numbers, whole numbers) results in a product larger than the given number.

5.NF.F.5C Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.

5.NF.F.5D Explain why multiplying the numerator and denominator by the same number has the same effect as multiplying the fraction by 1.

***5.NF.F.6 Solve real-world problems involving multiplication of fractions and mixed numbers by using visual fraction models or equations to represent the problem.**

The **Proficient** student is able to solve real-world problems involving multiplication of fractions and mixed numbers by using visual fraction models or equations to represent the problem.

5.NF.F.7 Extend the concept of division to divide unit fractions and whole numbers by using visual fraction models and equations.

5.NF.F.7A Interpret division of a unit fraction by a non-zero whole number and compute the quotient.

5.NF.F.7B Interpret division of a whole number by a unit fraction and compute the quotient.

5.NF.F.7C Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions by using visual fraction models and equations to represent the problem.

MEASUREMENT AND DATA

Convert like measurement units within a given measurement system.

5.MD.G.1 Understand a coordinate system.

Represent and interpret data.

5.MD.H.2 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions to solve problems involving information presented in line plots.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.I.3 Recognize volume as an attribute of three-dimensional figures and understand concepts of volume measurement such as "unit cube" and a volume of n cubic units.

5.MD.I.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and improvised units.

***5.MD.I.5 Relate volume to the operations of multiplication and solve real-world and mathematical problems involving volume.**

5.MD.I.5A Find the volume of a right rectangular prism with whole number dimensions by multiplying them. Show that this volume is the same as when counting unit cubes.

5.MD.I.5B Find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems given the formulas $V = (l)(w)(h)$ and $V = (B)(h)$ for rectangular prisms.

The **Proficient** student is able to relate volume to the operations of multiplication and solve real-world and mathematical problems involving volume.

A. Find the volume of a right rectangular prism with whole number dimensions by multiplying them. Show that this volume is the same as when counting unit cubes.

B. Find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems given the formulas $V = (l)(w)(h)$ and $V = (B)(h)$ for rectangular prisms.

GEOMETRY

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.J.1 Understand a coordinate system.

5.G.J.1A The x- and y- axes are perpendicular number lines that intersect at 0 (the origin).

5.G.J.1B Any point on the coordinate plane can be represented by its coordinates.

5.G.J.1C The first number in an ordered pair is the x-coordinate and represents the horizontal distance from the origin.

5.G.J.1D The second number in an ordered pair is the y-coordinate and represents the vertical distance from the origin.

5.G.J.2 Plot and interpret points in the first quadrant of the coordinate plane to represent real-world and mathematical situations.

Classify two-dimensional figures into categories based on their properties.

5.G.K.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

Assessment Boundary: Use polygons only.

5.G.K.4 Classify polygons in a hierarchy based on properties.

Grade 6 Math Content & Performance Standards

GRADE 6 MATH PRACTICES

MP1 Make sense of problems and persevere in solving them.

6.MP.1 In grade 6, students solve problems involving ratios and rates and discuss (verbally or in writing) how they solve them. Students analyze the problem (including what is given, not given, and what is being asked), identify what strategies are needed, recognize multiple pathways to a solution, and make an initial attempt to solve the problem. Students analyze the result for validity and refine strategies if necessary.

MP2 Reason abstractly and quantitatively.

6.MP.2 Students recognize a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students begin to contextualize to understand the meaning of the number or variable as it relates to the problem.

MP3 Construct viable arguments and critique the reasoning of others.

6.MP.3 Students begin to contextualize to understand the meaning of the number or variable as it relates to the problem. They make conjectures, explore validity, reason mathematically, justify, evaluate their own thinking.

MP4 Model with mathematics.

6.MP.4 Students can clearly show their work by using diagrams, words, symbols or pictures. They are able to identify important quantities in a practical situation and map their relationships using tools such as, diagrams, two-way tables, graphs, flowcharts or formulas. They can recognize and analyze those relationships mathematically to draw conclusions. They can interpret their mathematical results of problems involving non-negative rational numbers in the context of the situation and reflect on whether the results make sense.

MP5 Use appropriate tools strategically.

6.MP.5 Students consider available tools (including estimation, concrete models, and technology), and decide when certain tools might be helpful. They choose the representation (table, graph, equation, words) that best suits the problem. Students use concrete models to develop insight into ratios and other concepts. Students extend this insight to more abstract representations, including pictures and symbols. Students understand the limitations of each tool. Tools might include: unifix cubes, fraction bars, base-ten blocks, number lines, graph paper, calculator, paper and pencil, and others.

MP6 Attend to precision.

6.MP.6 Students continue to refine their mathematical communication and reasoning skills by using clear language in their discussions with others. students define variables, including their relationship, specify units of measure, and label each axis accurately. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities. Students use appropriate symbols, labels, and units of measure when solving problems with calculations that are accurate and efficient. The answer to the problem matches what was asked in the problem.

MP7 Look for and make use of structure.

6.MP.7 Students routinely seek patterns or structure to model and solve problems. They recognize that patterns exist in ratio tables. Students notice patterns and identify strategies for creating equivalent expressions. Students identify complicated expressions or figures as compositions of simple parts.

MP8 Look for and express regularity in repeated reasoning.

6.MP.8 Students use repeated reasoning to understand algorithms and make generalizations about patterns. They construct examples and models that confirm their generalization. They develop short cuts and check for

reasonableness of answers. Students ask questions such as, "How would we verify that?" and "How is this similar to patterns with whole numbers?

RATIOS AND PROPORTIONAL RELATIONSHIPS

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.A.2 Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

***6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems.**

6.RP.A.3A Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.RP.A.3B Solve unit rate problems including those involving unit pricing and constant speed.

6.RP.A.3C Understand that a percentage is a rate per 100 and use this to solve problems involving wholes, parts, and percentages.

6.RP.A.3D Use ratio reasoning to convert measurement units; convert units appropriately when multiplying or dividing quantities.

The **Proficient** student is able to:

A. Make tables of equivalent ratios relating quantities with whole number measurements and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
B. Solve unit rate problems with whole number measurements including those involving unit pricing and constant speed.
C. In mathematical and real-world contexts solve one-step problems involving wholes, parts, and percentages.
D. Use ratio reasoning to convert measurement units and to transform units appropriately when multiplying or dividing quantities in one-step problems.

THE NUMBER SYSTEM

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

***6.NS.B.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions by using visual fraction models and equations to represent the problem.**

The **Proficient** student is able to interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions by using visual fraction models and equations to represent the problem.

6.NS.B.2 Divide multi-digit numbers using efficient and generalizable procedures including, but not limited to the standard algorithm.

Assessment Boundary: Use up to 5-digit dividends, 2-digit divisors.

Compute fluently with multi-digit numbers and find common factors and multiples.

***6.NS.C.3** **Add, subtract, multiply, and divide manageable multi-digit decimals using efficient and generalizable procedures including, but not limited to the standard algorithm for each operation.**

The **Proficient** student is able to add, subtract, multiply, and divide multi-digit decimals using efficient and generalizable procedures including, but not limited to the standard algorithm for each operation.
Assessment Boundary: Limit decimals in the given values to the hundredths place.

6.NS.C.4 Find common factors and multiples using two whole numbers.

6.NS.C.4A Find the greatest common factor of two whole numbers less than or equal to 100.
6.NS.C.4B Find the least common multiple of two whole numbers less than or equal to 12.
6.NS.C.4C Use the Distributive Property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.D.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values and use them to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.D.6 Extend the understanding of the number line to include all rational numbers and apply this concept to the coordinate plane.

6.NS.D.6A Understand the concept of opposite numbers, including 0, and their relative locations on the number line.
6.NS.D.6B Understand that signs of numbers in ordered pairs indicate locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
6.NS.D.6C Find and position rational numbers on a horizontal or vertical number line diagram; find and position pairs of rational numbers on a coordinate plane.

6.NS.D.7 Understand ordering and absolute value of rational numbers.

6.NS.D.7A Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
6.NS.D.7B Write, interpret, and explain statements of order for rational numbers in real-world contexts.
6.NS.D.7C Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
6.NS.D.7D Distinguish comparisons of absolute value from statements about order.

***6.NS.D.8** **Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Find distances between points with the same first coordinate or the same second coordinate; relate absolute value and distance.**

The **Proficient** student is able to solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Find distances between points with the same first coordinate or the same second coordinate; relate absolute value and distance.

EXPRESSIONS AND EQUATIONS

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.E.1 Write and evaluate numerical expressions involving whole-number exponents.

***6.EE.E.2 Write, read, and evaluate expressions in which letters stand for numbers.**

6.EE.E.2A Write expressions that record operations with numbers and with letters standing for numbers.

6.EE.E.2B Identify parts of an expression using mathematical terms (sum, difference, term, product, factor, quotient, coefficient, constant).

6.EE.E.2C Use Order of Operations to evaluate algebraic expressions using positive rational numbers and whole-number exponents. Include expressions that arise from formulas in real-world problems.

The **Proficient** student is able to write, read, and evaluate expressions in which letters stand for numbers.

A. Write two-step algebraic expressions.

B. Identify parts of an expression using mathematical terms (sum, difference, term, product, factor, quotient, coefficient, constant).

C. Use Order of Operations to evaluate algebraic expressions using positive rational numbers and whole-number exponents. Include expressions that arise from formulas relative to sixth grade standards in real-world problems.

6.EE.E.3 Apply the properties of operations to generate equivalent expressions.

6.EE.E.4 Identify when two expressions are equivalent.

Reason about and solve one-variable equations and inequalities.

6.EE.F.5 Understand a solution to an equation or an inequality makes the equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.F.6 Use variables to represent unknown numbers and write expressions when solving a real-world or mathematical problem.

***6.EE.F.7 Write and solve real-world and mathematical problems in the form of one-step, linear equations involving nonnegative rational numbers.**

The **Proficient** student is able to solve problems in both real-world and mathematical contexts by writing and solving equations in the form of one-step, linear equations involving nonnegative rational numbers.

6.EE.F.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.G.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity (dependent variable), in terms of the other quantity (independent variable). Analyze their relationship using graphs and tables, and relate these to the equation.

GEOMETRY

Solve real-world and mathematical problems involving area, surface area, and volume.

***6.G.H.1 Find area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.**

The **Proficient** student is able to find area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.G.H.2 Find the volume of a right rectangular prism with fractional edge lengths in the context of solving real-world and mathematical problems by applying the formulas V = (l)(w)(h) and V = (B)(h), and label with appropriate units.

6.G.H.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

6.G.H.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures in the context of solving real-world and mathematical problems.

STATISTICS AND PROBABILITY

Develop understanding of statistical variability.

6.SP.I.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

6.SP.I.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

6.SP.I.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

6.SP.J.4 Display numerical data in plots on a number line, including dot plots, stem-and-leaf plots, histograms, and box plots.

***6.SP.J.5 Summarize numerical data sets in relation to their real-world context.**

6.SP.J.5A Report the sample size.

6.SP.J.5B Describe the context of the data under investigation, including how it was measured and its units of measurement.

6.SP.J.5C Find quantitative measures of center (median, mode and mean) and variability (range and interquartile range). Describe any overall pattern (including outliers, clusters, and distribution), with reference to the context in which the data was gathered.

6.SP.J.5D Justify the choice of measures of center (median, mode, or mean) based on the shape of the data distribution and the context in which the data was gathered.

The **Proficient** student is able to summarize numerical data sets in relation to their real-world context.

A. Report the sample size.
B. Describe the context of the data under investigation, including how it was measured and its units of measurement.
C. Find quantitative measures of center (median, mode, and mean) and variability (range and interquartile range). Describe any overall pattern (including outliers, clusters, and distribution), with reference to the context in which the data was gathered.
D. Justify the choice of measures of center (median, mode, or mean) based on the shape of the data distribution and the context in which the data was gathered.

Grade 7 Math Content & Performance Standards

GRADE 7 MATH PRACTICES

MP1 Make sense of problems and persevere in solving them.

7.MP.1 In grade 7, students solve real-world problems involving ratios, rates, proportions, rational numbers and geometric concepts and discuss (verbally or in writing) how they solve them. Students analyze the problem (including what is given, not given, and what is being asked), identify what strategies are needed, choose an appropriate pathway, then make an initial attempt to solve the problem. Students analyze the result for validity and refine strategies if necessary.

MP2 Reason abstractly and quantitatively.

7.MP.2 Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

MP3 Construct viable arguments and critique the reasoning of others.

7.MP.3 Students construct arguments using verbal or written explanations that involve solving problems with rational numbers. They make conjectures, explore validity, reason mathematically, justify, evaluate their own thinking and the thinking of other students.

MP4 Model with mathematics.

7.MP.4 Students can clearly show their work by using diagrams, words, symbols or pictures. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and/or formulas. They can analyze those relationships mathematically to draw conclusions. They interpret their mathematical results of problems involving rational numbers in the context of the situation and reflect on whether the results make sense.

MP5 Use appropriate tools strategically.

7.MP.5 Students consider available tools (including estimation, concrete models, and technology as appropriate), and decide when certain tools might be helpful. Students develop more efficacy with technology. They choose the representation (table, graph, equation, words) that best suits the problem. Students use concrete models to develop insight into proportions and other concepts. Students then extend this insight to more abstract representations, including pictures and symbols. Students understand the limitations of each tool. Tools might include: integer tiles, algebra tiles, geometric nets, number lines, graphing technology, scientific calculator, paper and pencil, and others.

MP6 Attend to precision.

7.MP.6 Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, including their relationship, specify units of measure, and label each axis accurately. Student use appropriate terminology when referring to rates, ratios, proportions, probability models, geometric figures, data displays, and components of expressions, equations or inequalities. Students use appropriate symbols, labels, and units of measure when solving problems with calculations that are accurate and efficient. Answer to the problem matches what was asked in the problem.

MP7 Look for and make use of structure.

7.MP.7 Students routinely seek patterns or structure to model and solve problems. They recognize that patterns exist in ratio tables and make connections with the constant of proportionality in a table and the slope of a graph. Students recognize patterns and identify and develop strategies for creating equivalent expressions. Students identify complicated expressions or figures as compositions of simple parts.MP8 Look for and express regularity in repeated reasoning.

MP8 Look for and express regularity in repeated reasoning.

7.MP.8 Students routinely seek patterns or structure to model and solve problems. They apply properties to solve problems based upon patterns they have identified. Students examine patterns to generate equations and describe relationships. Students simplify complicated expressions into simple terms. Students recognize the effects of transformations and describe them in terms of congruence and similarity.

RATIOS AND PROPORTIONAL RELATIONSHIPS

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.1 Compute unit rates, including those involving complex fractions, with like or different units.

***7.RP.A.2 Recognize and represent proportional relationships between quantities.**

7.RP.A.2A Decide whether two quantities in a table or graph are in a proportional relationship.
7.RP.A.2B Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
7.RP.A.2C Represent proportional relationships with equations.
7.RP.A.2D Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

The **Proficient** student is able to recognize and represent proportional relationships between quantities.

A. Decide whether two quantities in a table or graph are in a proportional relationship.
B. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
C. Represent proportional relationships with equations.
D. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

***7.RP.A.3 Solve multi-step real-world and mathematical problems involving ratios and percentages.**

The **Proficient** student is able to solve multi-step real-world and mathematical problems involving ratios and percentages (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error).

THE NUMBER SYSTEM

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.B.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers.

7.NS.B.1A Describe situations in which opposite quantities combine to make 0 (the additive identity).
7.NS.B.1B Understand that $p + q$ represents the distance $|q|$ from p whose placement is determined by the sign of q. Interpret sums of rational numbers by describing real-world contexts.
7.NS.B.1C Show that a number and its opposite have a sum of 0 (are additive inverses).
7.NS.B.1D Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Apply this principal in real-world contexts.
7.NS.B.1E Apply properties of addition as strategies to add and subtract rational numbers.

7.NS.B.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

7.NS.B.2A 1. Understand that the multiplicative inverse of a number is its reciprocal and their product is equal to one (the multiplicative identity). 2. Understand positive and negative sign rules for multiplying rational numbers. Interpret products of rational numbers by describing real-world contexts.

7.NS.B.2B Understand that integers can be divided, provided that the divisor is not 0, and every quotient of integers is a rational number. Recognize that if p and q are integers then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

7.NS.B.2C Apply properties of multiplication (Commutative, Associative, Distributive, or Properties of Identity and Inverse elements) to multiply and divide rational numbers.

7.NS.B.2D Convert a rational number to a decimal. Recognize that rational numbers can be written as fractions or decimal numbers that terminate or repeat.

***7.NS.B.3 Solve real-world and mathematical problems involving the four arithmetic operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)**

The **Proficient** student is able to solve real-world and mathematical problems involving the four arithmetic operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

EXPRESSIONS AND EQUATIONS

Use properties of operations to generate equivalent expressions.

7.EE.C.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.C.2 Recognize that algebraic expressions may have a variety of equivalent forms that reveal different information, and determine an appropriate form for a given real-world situation.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.D.3 Solve multi-step real-world and mathematical problems involving rational numbers. Include fraction bars as a grouping symbol.

***7.EE.D.4 Apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations.**

7.EE.D.4A Write and fluently solve linear equations of the form $ax + b = c$ *and* $a(x + b) = c$ where a, b, and c are rational numbers.

7.EE.D.4B Write and solve multi-step linear equations that include the use of the Distributive Property and combining like terms. Exclude equations that contain variables on both sides.

7.EE.D.4C Write and solve two-step linear inequalities. Graph the solution set on a number line and interpret its meaning.

7.EE.D.4D Identify and justify the steps for solving multi-step linear equations and two-step linear inequalities.

The **Proficient** student is able to apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations.

A. Write and fluently solve linear equations of the form $ax + b = c$ and $a(x + b) = c$ where a, b, and c are rational numbers.
B. Write and solve multi-step linear equations that include the use of the Distributive Property and combining like terms. Exclude equations that contain variables on both sides.
C. Write and solve two-step linear inequalities. Graph the solution set on a number line and interpret its meaning.
D. Identify and justify the steps for solving multi-step linear equations and two-step linear inequalities.

2021 Math Wyoming Content & Performance Standards

GEOMETRY

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.E.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing.

7.G.E.2 Draw geometric shapes with given conditions using a variety of tools (e.g., ruler and protractor, or technology). Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

7.G.E.3 Describe the two-dimensional figures that result from slicing three-dimensional figures parallel to the base, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

***7.G.F.4 Investigate the concept of circles.**

7.G.F.4A Demonstrate an understanding of the proportional relationships between diameter, radius, and circumference of a circle.
7.G.F.4B Understand that π is defined by the constant of proportionality between the circumference and diameter.
7.G.F.4C Given the formulas for circumference and area of circles, solve real-world and mathematical problems.

The **Proficient** student is able to investigate the concept of circles.

A. Demonstrate an understanding of the proportional relationships between diameter, radius, and circumference of a circle.
B. Understand that π is defined by the constant of proportionality between the circumference and diameter.
C. Given the formulas for circumference and area of circles, solve real-world and mathematical problems.

Assessment Boundary: Specify if calculations should be performed with 3.14 or the π button. Specify the place value to which answers must be rounded. Do not include solutions in terms of π.

7.G.F.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

***7.G.F.6 Solve real-world and mathematical problems involving.**

7.G.F.6A Area and surface area of objects composed of triangles and quadrilaterals;
7.G.F.6B Volume of objects composed only of right prisms having triangular or quadrilateral bases.

The **Proficient** student is able to solve real-world and mathematical problems involving:

A. Area and surface area of objects composed of triangles and quadrilaterals;
B. Volume of objects composed only of right prisms having triangular or quadrilateral bases.

STATISTICS AND PROBABILITY

Use random sampling to draw inferences about a population.

***7.SP.G.1 Solve real-world and mathematical problems involving:**

7.SP.G.1A Understand that a sample is a subset of a population.
7.SP.G.1B Differentiate between random and non-random sampling.
7.SP.G.1C Understand that generalizations from a sample are valid only if the sample is representative of the population.

7.SP.G.1D Understand that random sampling is used to gather a representative sample and tends to support valid inferences about the population.

The **Proficient** student is able to solve real-world and mathematical problems involving:

A. Describing a sample that is a subset of a population.
B. Differentiating between random and non-random sampling.
C. Determining if a generalization is valid by justifying whether or not the sample is representative of the population.
D. Determining if inferences about the population are valid based on how the given sample was collected.

7.SP.G.2 Draw inferences about a population by collecting multiple random samples of the same size to investigate variability in estimates of the characteristic of interest.

Draw informal comparative inferences about two populations.

7.SP.H.3 Visually compare the centers, spreads, and overlap of two displays of data (e.g., back-to-back stem and leaf plots, dot plots, histograms, box plots) that are graphed on the same scale and draw inferences about this data.

***7.SP.H.4** Given measures of center and variability (mean, median and/or mode; range, interquartile range, and/or standard deviation), for numerical data from random samples, draw appropriate informal comparative inferences about two populations.**

The **Proficient** student is able to given measures of center and variability (mean, median, and/or mode; range, interquartile range, and/or standard deviation), for numerical data from random samples, draw appropriate informal comparative inferences about two populations.

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.I.5 Find and interpret the probability of a random event. Understand that the probability of a random event is a number between, and including, 0 and 1 that expresses the likelihood of the event occurring.

7.SP.I.6 Collect multiple samples to compare the relationship between theoretical and experimental probabilities for simple events.

7.SP.I.7 Apply the concepts of theoretical and experimental probabilities for simple events.

7.SP.I.7A Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.
7.SP.I.7B Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.
7.SP.I.7C Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancies.

7.SP.I.8 Find probabilities of compound events using organized lists, tables, and tree diagrams.

7.SP.I.8A Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
7.SP.I.8B Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

Grade 8 Math Content & Performance Standards

GRADE 8 MATH PRACTICES

MP1 Make sense of problems and persevere in solving them.

8.MP.1 In grade 8, students solve real-world problems through the application of algebraic and geometric concepts and discuss (verbally or in writing) how they solve them. Students analyze the problem (including what is given, not given, and what is being asked), identify what strategies are needed, choose the most efficient pathway, then make an initial attempt to solve the problem. Students analyze the result for validity and refine strategies if necessary.

MP2 Reason abstractly and quantitatively.

8.MP.2 Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

MP3 Construct viable arguments and critique the reasoning of others.

8.MP.3 Students construct arguments using verbal or written explanations that involve solving problems with real numbers. They make conjectures, explore validity, reason mathematically, justify, evaluate their own thinking and analytically critique the reasoning of other students.

MP4 Model with mathematics.

8.MP.4 Students can clearly show their work by using diagrams, words, symbols or pictures. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results of problems involving real numbers in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5 Use appropriate tools strategically.

8.MP.5 Students consider available tools (including estimation, concrete models, and technology as appropriate), and decide when certain tools might be helpful. Students can interpret results provided by technology. They choose the representation (table, graph, equation, words) that best suits the problem. Students use concrete models to develop insight into linear equations and other concepts. Students then extend this insight to more abstract representations, including pictures and symbols. Students understand the limitations of each tool. Tools might include: integer tiles, algebra tiles, geometric nets, number lines, graphing technology, scientific calculator, paper and pencil, and others.

MP6 Attend to precision.

8.MP.6 Students continue to refine their mathematical communication skills by using clear and precise mathematical language in their discussions with others and in their own reasoning. Students define variables, including their relationship, specify units of measure, and label each axis accurately. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays. Students use appropriate symbols, labels, and units of measure when solving problems with calculations that are accurate and efficient. Answer to the problem matches what was asked in the problem.

MP7 Look for and make use of structure.

8.MP.7 Students routinely seek patterns or structure to model and solve problems. They apply properties to solve problems based upon patterns they have identified. Students examine patterns to generate equations and describe relationships. Students simplify complicated expressions into simple terms. Students recognize the effects of transformations and describe them in terms of congruence and similarity.

2021 Math Wyoming Content & Performance Standards

MP8 Look for and express regularity in repeated reasoning.

8.MP.8 Students use repeated reasoning to understand algorithms and make generalizations about patterns. They develop efficient strategies for solving problems and check for reasonableness of answers. Students ask questions such as, "What evidence supports that conclusion?"

THE NUMBER SYSTEM

Know that there are numbers that are not rational, and approximate them by rational numbers.

***8.NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. Explore the real number system and its appropriate usage in real-world situations.**

- **8.NS.A.1A** Make comparisons between rational and irrational numbers.
- **8.NS.A.1B** Understand that all real numbers have a decimal expansion.
- **8.NS.A.1C** Model the hierarchy of the real number system, including natural, whole, integer, rational, and irrational numbers.
- **8.NS.A.1D** Convert repeating decimals to fractions.

The **Proficient** student is able to know that numbers that are not rational are called irrational. Show that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. Explore the real number system and its appropriate usage in real-world situations.

A. Make comparisons between rational and irrational numbers.
B. Show that real numbers (excluding irrational numbers) have a decimal expansion.
C. Model the hierarchy of the real number system, including natural, whole, integer, rational, and irrational numbers.
D. Convert repeating decimals to fractions.

8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.

EXPRESSIONS AND EQUATIONS

Work with radicals and integer exponents.

8.EE.B.1 Understand and apply the Laws of Exponents (i.e., Product Rule, Quotient Rule, Power to a Power, Product to a Power, Quotient to a Power, Zero Power Property, negative exponents) to generate equivalent numerical expressions limited to integer exponents.

8.EE.B.2 Investigate concepts of square and cube roots.

Assessment Boundary: Include perfect squares up to 144 and perfect cubes up to 125.

- **8.EE.B.2A** Use radical notation, if applicable, to represent the exact solutions to equations of the form $x^2 = p$ and $x^3 = q$ where p is a positive rational number and q is any rational number.
- **8.EE.B.2B** Evaluate square roots of small perfect squares and cube roots of small perfect cubes.
- **8.EE.B.2C** Recognize that square roots of non-perfect squares and the cube roots of non-perfect cubes are irrational.

8.EE.B.3 Explore the relationship between quantities in decimal and scientific notation.

8.EE.B.3A Express very large and very small quantities, p, in scientific notation in the form $a \cdot 10^b = p$ where $1 \leq a < 10$ and b is an integer.
8.EE.B.3B Translate between decimal notation and scientific notation.
8.EE.B.3C Estimate and compare the relative size of two quantities in scientific notation.

8.EE.B.4 Apply the concepts of decimal and scientific notation to real-world and mathematical problems.
8.EE.B.4A Select appropriate units of measure when representing answers in scientific notation.
8.EE.B.4B Interpret scientific notation that has been generated by a variety of technologies.

Understand the connections between proportional relationships, lines, and linear equations.

***8.EE.C.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.**

The **Proficient** student is able to graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

8.EE.C.6 Explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y =mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $(0, b)$.

Analyze and solve linear equations and pairs of simultaneous linear equations.

***8.EE.D.7 Extend concepts of linear equations and inequalities in one variable to more complex multi-step equations and inequalities in real-world and mathematical situations.**
8.EE.D.7A Solve linear equations and inequalities with rational number coefficients that include the use of the Distributive Property, combining like terms, and variable terms on both sides.
8.EE.D.7B Recognize the three types of solutions to linear equations: one solution, infinitely many solutions, or no solutions.
8.EE.D.7C Generate linear equations with the three types of solutions.
8.EE.D.7D Justify why linear equations have a specific type of solution.

The **Proficient** student is able to extend concepts of linear equations and inequalities in one variable to more complex multi-step equations and inequalities in real-world and mathematical situations.

A. Solve linear equations and inequalities with rational number coefficients that include the use of the Distributive Property, combining like terms, and variable terms on both sides.
B. Recognize the three types of solutions to linear equations: one solution, infinitely many solutions, or no solutions.
C. Generate linear equations with the three types of solutions.
D. Justify why linear equations have a specific type of solution.

***8.EE.D.8 Analyze and solve pairs of simultaneous linear equations.**
8.EE.D.8A Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
8.EE.D.8B Solve systems of two linear equations in two variables with integer solutions by graphing the equations.
8.EE.D.8C Solve simple real-world and mathematical problems leading to two linear equations in two variables given $y = mx + b$ form with integer solutions.

The **Proficient** student is able to analyze and solve a system of linear equations.

A. Show that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously, including systems with one, infinitely many, and no solutions.

B. Solve systems of two linear equations in two variables with integer solutions by graphing the equations.
C. Solve simple real-world and mathematical problems leading to two linear equations in two variables given $y = mx + b$ form with integer solutions.

FUNCTIONS

Define, evaluate, and compare functions.

8.F.E.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

***8.F.E.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).**

The **Proficient** student is able to compare properties (intercepts, domain, and range) of two linear functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

8.F.E.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

Use functions to model relationships between quantities.

***8.F.F.4 Apply the concepts of linear functions to real-world and mathematical situations.**

8.F.F.4A Understand that the slope is the constant rate of change and the y-intercept is the point where $x = 0$.
8.F.F.4B Determine the slope and the y-intercept of a linear function given multiple representations, including two points, tables, graphs, equations, and verbal descriptions.
8.F.F.4C Construct a function in slope-intercept form that models a linear relationship between two quantities.
8.F.F.4D Interpret the meaning of the slope and the y-intercept of a linear function in the context of the situation.

The **Proficient** student is able to apply the concepts of linear functions to real-world and mathematical situations.
A. Recognize that the slope is the constant rate of change and the y-intercept is the point where $x = 0$ from an equation, graph, table, and verbal description.
B. Determine the slope and the y-intercept of a linear function given multiple representations, including two points, tables, graphs, equations, and verbal descriptions.
C. Construct a function in slope-intercept form that models a linear relationship between two quantities.
D. Interpret the meaning of the slope and the y-intercept of a linear function in the context of the situation.

8.F.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph where the function is increasing, decreasing, constant, linear, or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

GEOMETRY

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.G.1 Verify experimentally the properties of rotations, reflections, and translations.

8.G.G.1A Lines are taken to lines, and line segments to line segments of the same length.

8.G.G.1B Angles are taken to angles of the same measure.
8.G.G.1C Parallel lines are taken to parallel lines.

8.G.G.2 Recognize through visual comparison that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.G.3 Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.

8.G.G.4 Recognize through visual comparison that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

8.G.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Understand and apply the Pythagorean Theorem.

8.G.H.6 Use models or diagrams to explain the Pythagorean Theorem and its converse.

***8.G.H.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems.**

The **Proficient** student is able to apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems.

8.G.H.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

8.G.I.9 Given the formulas, solve real-world and mathematical problems involving volume and surface area of cylinders.

STATISTICS AND PROBABILITY

Investigate patterns of association in bivariate data.

8.SP.J.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe the association by form (linear / nonlinear), direction (positive / negative), strength (correlation), and unusual features.

8.SP.J.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.J.3 Use an equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

8.SP.J.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

8.SP.J.4A Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.

8.SP.J.4B Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

High School Math Content & Performance Standards

GRADE HS MATH PRACTICES

MP1 Make sense of problems and persevere in solving them.

HS.MP.1 Students start to examine problems by explaining to themselves the meaning of a problem and restating the problem in their own words. These students analyze the given information in the problem, including constraints, relationships, and goals. Students make conjectures about the form and meaning of the solution, devise a plan, and solve. They will consider both similar problems, and simpler forms of the original problem, in order to gain insight and efficiency in problem solving. Students monitor and evaluate their progress and change course if necessary. Students may utilize algebraic methods or technology. Students explain relationships between equations and the following: descriptions/situations, tables, and graphs. Students produce diagrams of important features and relationships, graph data, and search for patterns or trends. They check answers to problems and continually ask if the solution makes sense in context. They understand different approaches to solving complex problems and identify correspondences between different approaches.

MP2 Reason abstractly and quantitatively.

HS.MP.2 Students seek to make sense of quantities and explore relationships in problem situations. Students represent a given situation by defining and manipulating variables. Students consider the units involved and attend to the meaning of quantities in addition to computational reasoning -- knowing and using the different properties of operations.

MP3 Construct viable arguments and critique the reasoning of others.

HS.MP.3 Students understand and use stated assumptions, definitions, and previously established results in constructing arguments. Students make conjectures and build logical progressions of statements to explore the truth of their conjectures. They are able to analyze situations through decomposition and produce counterexample(s) if necessary. Students justify their conclusions, communicate these conclusions, and respond to arguments of others. Students make plausible arguments by reasoning inductively about the data and take into account the context from which the data arose. Students are able to compare the effectiveness of two plausible arguments, and distinguish correct logic from flawed logic. If there is a flaw in an argument, then they explain why the logic is flawed. Students determine a general process and/or domain to which an argument applies. The students listen or read the arguments of others, decide whether the argument makes sense, and ask useful questions to clarify or improve the arguments.

MP4 Model with mathematics.

HS.MP.4 Students apply their mathematical knowledge to solve problems arising in everyday life, society, and the workplace. Students may use geometry to solve a design problem or they may use a function to describe how one quantity of interest depends on another. Students may use assumptions and approximations to simplify a complicated situation and realize these may need revision later. Students identify important relationships between quantities in a practical situation and map these relationships using tools such as: diagrams, two-way tables, graphs, flowcharts, and formulas. Students analyze those relationships mathematically to draw conclusions and interpret the results in the context of the situation. Students are reflective of the results and may improve the model if it has not served the purpose.

MP5 Use appropriate tools strategically.

HS.MP.5 Students consider appropriate tools when solving a mathematical problem, including but not limited to: a) pencil and paper, b) concrete models, c) ruler, d) protractor, e) calculator, f) spreadsheet, and g) analytical software applications. Students familiar with mathematical tools make sound decisions about when each of these tools may be helpful and recognize both the insight to be gained and the limitations of the tool. Students may use a graphing calculator to analyze graphs of functions knowing that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Students may identify relevant external mathematical resources, such as digital content located on

a website, and use those resources to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6 Attend to precision.

HS.MP.6 Students communicate using mathematically correct definitions in their own reasoning and in discussions with others. They state the meaning of symbols they choose, specify units of measure, and label axes in order to clarify the correspondence with quantities in a problem. Students accurately and efficiently calculate. They express numerical answers with the degree of precision appropriate for the problem context.

MP7 Look for and make use of structure.

HS.MP.7 Students look closely to discern a pattern or structure and holistically consider the overview. Students may shift perspectives if needed to gain understanding of the pattern or structure. Students in algebra may use patterns to create equivalent expressions, factor and solve equations, compose functions, and transform figures. They may consider certain algebraic expressions as single objects or as being composed of several objects. Students in geometry recognize the significance of an existing line in a geometric figure and may use the strategy of drawing an auxiliary line for solving problems.

MP8 Look for and express regularity in repeated reasoning.

HS.MP.8 Students notice repeated calculations, look for general expressions to annotate the calculation, and consider potential shortcuts. Students maintain oversight of a process as they work to solve problems, derive formulas, or make generalizations, while attending to details. They assess the reasonableness of their intermediate results.

NUMBER AND QUANTITY

THE REAL NUMBER SYSTEM

Extend the properties of exponents to rational exponents.

N.RN.A.1 Explain how the meaning of the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

***N.RN.A.2** **Rewrite expressions involving radicals and rational exponents using the properties of exponents.**

The **Proficient** student is able to rewrite expressions involving radicals and rational exponents, using the properties of exponents.

Use properties of rational and irrational numbers.

N.RN.B.3 Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

QUANTITIES

Reason quantitatively and use units to solve problems.

***N.Q.C.1** **Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; and choose and interpret the scale and the origin in graphs and data displays.**

The **Proficient** student is able to use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; **and** choose and interpret the scale and the origin in graphs and data displays.

***N.Q.C.2** **Define appropriate quantities for the purpose of descriptive modeling.**

The **Proficient** student is able to define appropriate quantities for the purpose of descriptive modeling.

N.Q.C.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

THE COMPLEX NUMBER SYSTEM

Perform arithmetic operations with complex numbers.

N.CN.D.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

N.CN.D.2 Use the relation $i^2 = -1$ and the Commutative, Associative, and Distributive Properties to add, subtract, and multiply complex numbers.

N.CN.D.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

N.CN.E.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N.CN.E.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + i\sqrt{3})^3 = 8$ because $(-1 + i\sqrt{3})$ has modulus 2 and argument 120°.

N.CN.E.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

***N.CN.F.7** **Solve quadratic equations with real coefficients that have complex solutions.**

The **Proficient** student is able to solve quadratic equations with real coefficients that have complex solutions.

N.CN.F.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

N.CN.F.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

VECTOR AND MATRIX QUANTITIES

Represent and model with vector quantities.

N.VM.G.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $v, |v|, ||v||, \mathbf{v}$).

N.VM.G.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

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***N.VM.G.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.**

(+) The **Proficient** student is able to solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

***N.VM.H.4 (+) Add and subtract vectors.**

N.VM.H.4A Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

N.VM.H.4B Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

N.VM.H.4C Understand vector subtraction $v - w$ as $v + (-w)$, where $(-w)$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

(+) The **Proficient** student is able to add and subtract vectors.

A. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

B. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

C. Understand vector subtraction $v - w$ as $v + (-w)$, where $(-w)$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N.VM.H.5 (+) Multiply a vector by a scalar.

N.VM.H.5A Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise.

N.VM.H.5B Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

Perform operations on matrices and use matrices in applications.

***N.VM.I.6 (+) Use matrices to represent and manipulate data.**

(+) The **Proficient** student is able to use matrices to represent and manipulate data.

N.VM.I.7 (+) Multiply matrices by scalars to produce new matrices.

N.VM.I.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.

N.VM.I.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the Associative and Distributive Properties.

N.VM.I.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N.VM.I.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

N.VM.I.12 (+) Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

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ALGEBRA

SEEING STRUCTURE IN EXPRESSIONS

Interpret the structure of expressions.

A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

A.SSE.A.1A Interpret parts of an expression, such as terms, factors, and coefficients.

A.SSE.A.1B Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

Write expressions in equivalent forms to solve problems.

***A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.**

A.SSE.B.3A Factor a quadratic expression to reveal the zeros of the function it defines.

A.SSE.B.3B Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

A.SSE.B.3C Use the properties of exponents to transform expressions for exponential functions. Apply the concepts of decimal and scientific notation to solve real-world and mathematical problems.

i. Multiply and divide numbers expressed in both decimal and scientific notation.

ii. Add and subtract numbers in scientific notation with the same integer exponent.

The **Proficient** student is able to choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A. Factor a quadratic expression to reveal the zeros of the function it defines.

B. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

C. Use the properties of exponents to transform expressions for exponential functions. Apply the concepts of decimal and scientific notation to solve real-world and mathematical problems.

I. Multiply and divide numbers expressed in both decimal and scientific notation.

II. Add and subtract numbers in scientific notation with the same integer exponent.

A.SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS

Perform arithmetic operations on polynomials.

***A.APR.C.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.**

The **Proficient** student is able to understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

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Understand the relationship between zeros and factors of polynomial.

A.APR.D.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by $(x - a)$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

***A.APR.D.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.**

The **Proficient** student is able to identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

A.APR.E.4 Prove polynomial identities and use them to describe numerical relationships.

A.APR.E.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rewrite rational expressions.

A.APR.F.6 Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using inspection, long division, or, for the more complicated examples, a computer algebra system. (i.e., rewriting a rational expression as the quotient plus the remainder over divisor).

A.APR.F.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

CREATING EQUATIONS

Create equations that describe numbers or relationships.

***A.CED.G.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**

The **Proficient** student is able to create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

***A.CED.G.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.**

The **Proficient** student is able to create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

***A.CED.G.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.**

The **Proficient** student is able to represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

A.CED.G.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

REASONING WITH EQUATIONS AND INEQUALITIES

Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.H.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

***A.REI.H.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.**

The **Proficient** student is able to solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

***A.REI.I.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.**

The **Proficient** student is able to solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

***A.REI.I.4 Solve quadratic equations in one variable.**

A.REI.I.4A Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.

A.REI.I.4B Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

A.REI.I.4C (+) Derive the quadratic formula from the general form of a quadratic equation.

The **Proficient** student is able to solve quadratic equations in one variable.

A. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.

B. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

C. (+) Derive the quadratic formula from the general form of a quadratic equation.

Solve systems of equations.

A.REI.J.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

***A.REI.J.6 Estimate solutions graphically and determine algebraic solutions to linear systems, focusing on pairs of linear equations in two variables.**

The **Proficient** student is able to estimate solutions graphically and determine algebraic solutions to linear systems, focusing on pairs of linear equations in two variables.

***A.REI.J.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.**

The **Proficient** student is able to solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

A.REI.J.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.

A.REI.J.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Represent and solve equations and inequalities graphically.

A.REI.K.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

A.REI.K.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

A.REI.K.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

FUNCTIONS

INTERPRETING FUNCTIONS

Understand the concept of a function and use function notation.

***F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.**

The **Proficient** student is able to demonstrate that a function's domain is assigned to exactly one element of the range in equations, tables, graphs, and context.

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Interpret functions that arise in applications in terms of the context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

***F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.**

F.IF.C.7A Graph linear and quadratic functions and show intercepts, maxima, and minima.
F.IF.C.7B Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
F.IF.C.7C Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
F.IF.C.7D (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
F.IF.C.7E Graph exponential and logarithmic functions, showing intercepts and end behavior.
F.IF.C.7F (+) Graph trigonometric functions, showing period, midline, and amplitude.

The **Proficient** student is able to graph linear, quadratic, and exponential functions expressed symbolically and show appropriate key features of the graph showing intercepts, maxima, and minima, and end behavior.
A. Graph linear and quadratic functions and show intercepts, maxima, and minima.
B. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
C. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
D. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
E. Graph exponential and logarithmic functions, showing intercepts and end behavior
F. (+) Graph trigonometric functions, showing period, midline, and amplitude.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.C.8A Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
F.IF.C.8B Use the properties of exponents to interpret expressions for exponential functions.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

BUILDING FUNCTIONS

Build a function that models a relationship between two quantities.

***F.BF.D.1 Write a function that describes a relationship between two quantities.**

F.BF.D.1A Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.BF.D.1B Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
F.BF.D.1C (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

The **Proficient** student is able to write a function that describes a relationship between two quantities.
A. Determine an explicit expression, a recursive process, or steps for calculation from a context.
B. Combine standard function types using arithmetic operations.
C. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

F.BF.D.2 (+) Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

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Build new functions from existing functions.

***F.BF.E.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.**

The **Proficient** student is able to identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.BF.E.4 Find inverse functions.

F.BF.E.4A Write an expression for the inverse of a simple, invertible function $f(x)$. Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, as f and g are inverse functions, if and only if, $f(x) = y$ and $g(y) = x$, for all values of x in the domain of f and all values of y in the domain of g.

F.BF.E.4B (+) Verify by composition that one function is the inverse of another.

F.BF.E.4C (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

F.BF.E.4D (+) Produce an invertible function from a non-invertible function by restricting the domain.

F.BF.E.5 (+) Build new functions from existing functions. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

LINEAR, QUADRATIC, AND EXPONENTIAL MODELS

Construct and compare linear, quadratic, and exponential models and solve problems.

***F.LE.F.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.**

F.LE.F.1A Verify that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

F.LE.F.1B Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F.LE.F.1C Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

The **Proficient** student is able to distinguish between situations that can be modeled with linear functions and with exponential functions.

A. Verify that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

B. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

C. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

***F.LE.F.2 Construct linear and exponential functions using a graph, a description of a relationship, or two input-output pairs (include reading these from a table).**

The **Proficient** student is able to construct linear and exponential functions using a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.F.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.LE.F.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a, c,$ and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.

F.LE.G.5 Interpret the parameters in a linear or exponential function in terms of a context.

TRIGONOMETRIC FUNCTIONS

Extend the domain of trigonometric functions using the unit circle.

***F.TF.H.1 (+) Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.**

(+) The **Proficient** student is able to demonstrate that radian measure of an angle is the length of the arc on the unit circle subtended by the angle.

F.TF.H.2 (+) Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

***F.TF.H.3 (+) Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.**

(+) The **Proficient** student is able to use special triangles to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

F.TF.H.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

***F.TF.I.5 (+) Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.**

(+) The **Proficient** student is able to choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F.TF.I.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F.TF.I.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.

F.TF.J.8 (+) Prove the Pythagorean identity $sin^2 A + cos^2 A = 1$ and use it to find $sin\, A$, $cos\, A$, or $tan\, A$, given $sin\, A$, $cos\, A$, or $tan\, A$, and the quadrant of the angle.

F.TF.J.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

2021 Math Wyoming Content & Performance Standards

GEOMETRY

CONGRUENCE

Experiment with transformations in the plane.

***G.CO.A.1 Apply precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.**

The **Proficient** student is able to apply precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G.CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G.CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

***G.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.**

The **Proficient** student is able to, given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

G.CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G.CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems.

G.CO.C.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

***G.CO.C.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.**

The **Proficient** student is able to prove theorems about triangles. Theorems include: measure of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

G.CO.C.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Make geometric constructions.

G.CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY

Understand similarity in terms of similarity transformations.

G.SRT.E.1 Verify heuristically the properties of dilations given by a center and a scale factor.

G.SRT.E.1A A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

G.SRT.E.1B The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G.SRT.E.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G.SRT.E.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity.

G.SRT.F.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

***G.SRT.F.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.**

The **Proficient** student is able to use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles.

G.SRT.G.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G.SRT.G.7 Explain and use the relationship between the sine and cosine of complementary angles.

***G.SRT.G.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.**

The **Proficient** student is able to use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Apply trigonometry to general triangles.

G.SRT.H.9 (+) Derive the formula $A = \frac{1}{2}ab\,sin(c)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G.SRT.H.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G.SRT.H.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

CIRCLES

Understand and apply theorems about circles.

G.C.I.1 Prove that all circles are similar.

***G.C.I.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.**

The **Proficient** student is able to identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

G.C.I.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

G.C.I.4 (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles.

G.C.J.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS

Translate between the geometric description and the equation for a conic section.

G.GPE.K.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.GPE.K.2 (+) Derive the equation of a parabola given a focus and directrix.

G.GPE.K.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Use coordinates to prove simple geometric theorems algebraically.

G.GPE.L.4 Use coordinates to prove simple geometric theorems algebraically.

***G.GPE.L.5** **Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).**

The **Proficient** student is able to prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G.GPE.L.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

***G.GPE.L.7** **Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (e.g., using the distance formula).**

The **Proficient** student is able to use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).

GEOMETRIC MEASUREMENT AND DIMENSION

Explain volume formulas and use them to solve problems.

G.GMD.M.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

G.GMD.M.2 (+) Give an informal argument using Cavalieri's Principle for the formulas for the volume of a sphere and other solid figures.

***G.GMD.M.3** **Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.**

The **Proficient** student is able to use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Visualize relationships between two-dimensional and three-dimensional objects.

G.GMD.M.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

MODELING WITH GEOMETRY

Apply geometric concepts in modeling situations.

G.MG.O.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

***G.MG.O.2** **Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).**

The **Proficient** student is able to apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

***G.MG.O.3** **Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).**

The **Proficient** student is able to apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

STATISTICS AND PROBABILITY

INTERPRETING CATEGORICAL AND QUANTITATIVE DATA

Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots) by hand or using technology.

***S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.**

The **Proficient** student is able to use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S.ID.A.4 (+) Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use the Empirical Rule, calculators, spreadsheets, and/or tables to estimate areas under the normal curve.

S.ID.B.5 (+) Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations in the data, and use inferential statistical techniques to show association.

***S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.**

S.ID.B.6A Use a function to describe data trends to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

S.ID.B.6B (+) Informally assess the fit of a function by plotting and analyzing residuals.

S.ID.B.6C Using technology, fit a least squares linear regression function for a scatter plot that suggests a linear association.

The **Proficient** student is able to represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

A. Use a function to describe data trends to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

B. (+) Informally assess the fit of a function by plotting and analyzing residuals.

C. Using technology, fit a least squares linear regression function for a scatter plot that suggests a linear association.

Interpret linear models.

***S.ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.**

The **Proficient** student is able to interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S.ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

S.ID.C.9 Distinguish between correlation and causation.

MAKING INFERENCES AND JUSTIFYING CONCLUSIONS

Understand and evaluate random processes underlying statistical experiments.

S.IC.D.1 (+) Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.IC.D.2 (+) Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.E.3 (+) Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

***S.IC.E.4 (+) Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.**

(+) The **Proficient** student is able to use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S.IC.E.5 (+) Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

***S.IC.E.6 (+) Evaluate reports based on data.**

(+) The **Proficient** student is able to (+) evaluate reports based on data.

CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY

Understand independence and conditional probability and use them to interpret data.

***S.CP.F.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").**

The **Proficient** student is able to describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

***S.CP.F.2 (+) Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.**

(+) The **Proficient** student is able to understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S.CP.F.3 (+) Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S.CP.F.4 (+) Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

***S.CP.F.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.**

The **Proficient** student is able to recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.G.6 (+) Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

S.CP.G.7 (+) Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

S.CP.G.8 (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = [P(A)]x[P(B|A)] = [P(B)]x[P(A|B)]$, and interpret the answer in terms of the model.

S.CP.G.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

USE PROBABILITY TO MAKE DECISIONS

Calculate expected values and use them to solve problems.

S.MD.H.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

S.MD.H.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

***S.MD.H.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.**

(+) The **Proficient** student is able to develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.

S.MD.H.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

Use probability to evaluate outcomes of decisions.

***S.MD.I.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.**

S.MD.I.5A Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.

S.MD.I.5B Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

(+) The **Proficient** student is able to weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

A. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.

B. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

Calculate expected values and use them to solve problems.

S.MD.I.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

***S.MD.I.7** **(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).**

(+) The **Proficient** student is able to analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).