Facilitator’s Guide – Fraction Progressions (CCSS-M)
WDE Contact: Laurie.Hernandez@wyo.gov
Finalized on February 6, 2014

Who
• Grade 2-6 Teachers in PLCs

Why
• To gain an understanding of the fraction progressions across a number of grade levels informed by research on children’s cognitive development and the structure of mathematics.
• To build collaboration / articulation within and across grades.
• To further develop professional learning using additional resources by grade levels.

When
• In PLC’s or building groups (not intended to be a ‘one and done’)
• Three - four hours with the possibility of breaking it up.

What (Content)
• Meaning and Modeling of fractions
• Comparing and equivalencies of fractions
• Operations with fractions

What for (Achievement-Based Objectives)
• To build content knowledge and understanding of the progressions through examination of a major topic across a number of grade levels informed by research on children’s cognitive development and the structure of mathematics.
• To build collaboration/articulation within and across grades.

How (Learning Tasks)
• Begin with learning experience embedded in math…a math task on fractions that would lead to unpacking the progression document.
• Embed resources in the PD for use throughout the year.
• Pulling out salient points from the document, not reading the document.
The following structure does not include the following elements; you may want to include:

- Introduction
- Norms
- Breaks or Meals

**Materials Needed**

- Paper, pens/pencils, highlighters
- Chart Paper for small group share out
- Access to You Tube and Internet
- Speakers/System (connected to your computer so the videos can be heard by the group)

**Copies (1/participant) - (printed and/or *electronic)**

- Fraction Progressions Overview (copy at the end of this guide)
- Slide 3 – Shifts in Mathematical Instruction (copy at the end of this guide)
- Slide 7 – Fraction Progressions (copy at the end of this guide)
- Slide 14 (copy at the end of this guide)
- Cookie Activity (copy at the end of this guide)
- Reflection of Conceptual Understanding (copy at the end of this guide)
- *Progression Documents – [http://ime.math.arizona.edu/progressions](http://ime.math.arizona.edu/progressions)*
- Slide 7 - Blow up of Progressions Table – *optional* (can use chart paper, whiteboard, or smart board)

**Icon Key**

- = Say to your audience
- = Show Video
- = Reading & Writing Activity
- = Writing Activity
- = Group Discussion (small or large)
- = Next Slide

The following training contains 8 videos. You may decide, based on your audience, to show all of them or just a select few. Please feel free to adjust as needed.

**Total Time: Approximately 4 hours**

**Possible Breakdown of Time for 1 hour bits:**

Slides 1-12, 13-19, 20-31, 32-40 (time is estimated with all Videos & Activities)

Please consider doing a deep dive into this subject matter as it is a major shift for 3rd – 5th grade.
<table>
<thead>
<tr>
<th>Timing / Slide #s</th>
<th>Title of Activity, Document, or Link to Webpage</th>
<th>Icon Direction</th>
<th>Facilitator’s Information / Directions to Participants</th>
</tr>
</thead>
</table>
| 5 min. Slide 2   | Agenda                                        |                | - Objectives of Presentation  
- Fraction Overview  
- The Meaning of Fractions  
- Equivalent Fractions  
- Comparing Fractions  
- Operations with Fractions  
- Gain an understanding of the fraction progressions across grades 2-6, informed by research on children’s cognitive development and the structure of mathematics.  
- Collaborate within and across grades.  
- Further develop professional learning using additional resources by grade level. |
| 1 min. Slide 3   | Objectives                                    |                | *You’ll need to perform 3 clicks.  
- What type of **FOCUS** do I need in my grade level to help a student be successful on a problem such as this?  
- How do we work together within AND across grade levels to ensure **COHERENCE**?  
- How do we maintain proper **RIGOR** in our instruction including: Conceptual Understanding, Fluency, and Application? |
| 15 min. Slide 4   | Key ?s for this Training                      |                | Please refer to the Fraction Progressions Overview document. (15 min.)  
- Please read individually.  
- Underline the sentences which you think are the most important in unit development.  
- Share your sentences with the group. (Facilitator can decide if share out is with small groups or large group). |
<p>| 15 min. Slide 5   | Fraction Overview                             |                |                                                     |</p>
<table>
<thead>
<tr>
<th>Timing / Slide #s</th>
<th>Title of Activity, Document, or Link to Webpage</th>
<th>Icon</th>
<th>Facilitator’s Information / Directions to Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 min. Slide 6</td>
<td>The Overview of the Common Core Fractions Progressions Model</td>
<td><img src="http://youtu.be/X9NFEZlkoH0" alt="Image" /></td>
<td>Show video of the Overview. (3 min.)</td>
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<tr>
<td>15 min. Slide 7-9</td>
<td>Activity &amp; Fraction Progressions (copy is at the end of guide)</td>
<td><img src="http://vimeo.com/66775207#=0" alt="Image" /></td>
<td>Using your <strong>CCSS-M Document</strong>, work in pairs to identify the fraction standards in grades 2, 3, 4, or 5 and complete the Fraction Progressions Table. Complete chart in pairs by labeling the coding that fits. (i.e. 2.G.1 = grade.domain.cluster. The shaded boxes are to be left blank. Project Slide 7 and hand out copy of the <em>Fraction Progressions</em>. (5 min.) Share your findings with the group. (10 min.) Advance to Slide 8 to show filled in copy of chart.</td>
</tr>
<tr>
<td>2 min. Slide 10</td>
<td>Where are the Cookies?</td>
<td><img src="http://vimeo.com/66775207#=0" alt="Image" /></td>
<td>Keep the following problem in mind as we work through today’s activities. We will revisit it at the end of this training.</td>
</tr>
<tr>
<td>16 min. Slide 11</td>
<td>Meaning of Unit Fractions <a href="http://vimeo.com/66775207#=0">http://vimeo.com/66775207#=0</a></td>
<td><img src="http://vimeo.com/66775207#=0" alt="Image" /></td>
<td>Show video on Unit Fractions. (6 min.) <em>Read the section of the Progressions Document on development of the meaning of fractions and the number line.</em></td>
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<td>Icon Direction</td>
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<td>• Work in pairs to answer the following question: (10 min.)</td>
<td>![People Icon]</td>
<td>- What are the important aspects of fractions that provide opportunities for mathematical practice of attending to precision?</td>
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<tr>
<td>10 min.</td>
<td>Specify the Whole</td>
<td>![Microphone Icon]</td>
<td>Read the gray boxes titled <em>The Importance of Specifying the Whole</em> AND <em>Area Representations of ¼</em>.</td>
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<tr>
<td>Slide 13</td>
<td>Discussion</td>
<td>![People Icon]</td>
<td>• What is meant by “equal” parts?</td>
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<td>Specifying the Whole Discussion</td>
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<tr>
<td>15 min.</td>
<td>Equivalent Fractions</td>
<td>![Play Icon]</td>
<td>Show video on equivalent fractions. (5 min.)</td>
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<tr>
<td>Slide 14</td>
<td><a href="http://vimeo.com/68240897#at=0">http://vimeo.com/68240897#at=0</a></td>
<td>![Pen Icon]</td>
<td>Hand out copy <em>Equivalent Fractions</em> and have them work individually on the problem. (5 min.)</td>
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<tr>
<td></td>
<td>Equivalent Fractions Activity (copy is at the end of guide)</td>
<td>![Arrow Icon]</td>
<td>Advance slide to show an example of a completed model.</td>
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<tr>
<td>Slides 15-16</td>
<td>Discussion</td>
<td>![People Icon]</td>
<td>After watching the video and doing the activity, how has your perception of equivalent fractions and creating experiences for students about equivalent fractions changed? (5 min.)</td>
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<td>Slide 17</td>
<td>Comparing Fractions</td>
<td>![Play Icon]</td>
<td>Show video on comparing fractions. (7 min.)</td>
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<td>7 min.</td>
<td><a href="http://vimeo.com/68253451#at=0">http://vimeo.com/68253451#at=0</a></td>
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<td>20 min. Slide 19</td>
<td>Which Fraction is Larger?</td>
<td>Microphone</td>
<td>Which fraction is larger? Note: The denominators in this problem are for the adults, not the students. Note: Depending on your group size, decide if each person does a, b, &amp; c or if you break it out by groups. (5-10 min.)</td>
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</table>
| Slide 20         | Activity                                      | Book           | • Read the section of the Progressions Document on Grade 4 Equivalent Fractions.  
• Work in pairs to answer the following question:  
  • How can the use of area models and number line diagrams solidify a student’s understanding of fraction comparison? (10 min.)  |
| 20 min. Slide 21 | Adding Fractions                             | Play Video     | Show video on adding fractions. (6 min.)  |
| Slide 22         | Activity                                      | Pencil         | Project slide and work through. (3 min.) [poss. use of chart paper]  |
| Slides 23-24     | Solutions                                    | People         | Discuss various approaches to this problem. Have participants display on chart paper.  |
| Slide 25         | Activity                                      | Book           | Advance slide (animated: 4 additional clicks) to see one possible approach and to next slide (animated: 3 additional clicks) to see the final solution. *Please point out that we have a unit (12/12 = 1) + a part of a unit.  
• Read the section of the Progressions Document on Grade 4 and Grade 5 Adding and Subtracting Fractions.  |
<table>
<thead>
<tr>
<th>Timing / Slide #s</th>
<th>Title of Activity, Document, or Link to Webpage</th>
<th>Icon Direction</th>
<th>Facilitator’s Information / Directions to Participants</th>
</tr>
</thead>
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<td>• Work in pairs to answer the following question:</td>
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<td>• How could a student build on their previous knowledge of adding/subtracting whole numbers in order to add/subtract fractions? (5 min.)</td>
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<td>• Work in pairs to,</td>
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<td>• Discuss how multiplying a fraction by a whole number is similar to/different from multiplying whole numbers.</td>
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<td>• Discuss some of the misconceptions students may have when multiplying a fraction by a whole number. (10 min.)</td>
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<td>15 min.</td>
<td>Slide 26 Multiplying Fractions (part 1) <a href="http://vimeo.com/71857774">http://vimeo.com/71857774</a></td>
<td><img src="play.png" alt="Icon" /></td>
<td>Show video on Multiplying Fractions (part 1). (4 min.)</td>
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<td>Slide 27 Questions for Discussion</td>
<td><img src="pairs.png" alt="Icon" /></td>
<td>Work in pairs to,</td>
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<td>• Discuss how multiplying a fraction by a whole number is similar to/different from multiplying whole numbers.</td>
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<td>• Discuss some of the misconceptions students may have when multiplying a fraction by a whole number. (10 min.)</td>
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<td>16 min.</td>
<td>Slide 28 Multiplying Fractions (part 2) <a href="http://vimeo.com/71859340#at=0">http://vimeo.com/71859340#at=0</a></td>
<td><img src="play.png" alt="Icon" /></td>
<td>Show video on Multiplying Fractions (part 2). (6 min.)</td>
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<td>Slide 29 Questions for Discussion</td>
<td><img src="pairs.png" alt="Icon" /></td>
<td>Optional stopping point to process and discuss. (2:22)</td>
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<td>Work in pairs to,</td>
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<td>• Discuss one advantage and one disadvantage of using an area model when multiplying two fractions.</td>
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<td>• Create an area model that justifies each of your responses. (10 min.)</td>
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<td>8 min. Slide 30</td>
<td>Questions for Discussion</td>
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<td>With your partners, discuss how transparencies and color markers can be used to model the problem below: $\frac{3}{4} \times \frac{2}{5} = \frac{3\times2}{4\times5}$ (5 min.) Project slide for a demonstration of one possible solution.</td>
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<td>8 min. Slide 31</td>
<td>Possible Solution</td>
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<td>10 min. Slide 32</td>
<td>Activity</td>
<td></td>
<td>• Read the section of the Progressions Document on Grade 4 and Grade 5 Multiplying and Dividing Fractions. (10 min.) • Work with a partner to respond to the following item: • Explain how creating a story/real-world context might assist a student in understanding fraction multiplication. (5 min.)</td>
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<tr>
<td>5 min. Slide 33</td>
<td>Dividing Fractions</td>
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<td>Show video on Dividing Fractions. (5 min.)</td>
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<td><a href="http://vimeo.com/71907763#at=0">http://vimeo.com/71907763#at=0</a></td>
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<tr>
<td>15 min. Slide 34</td>
<td>50 Pounds of Rice</td>
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<td>If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Using a visual model, show how each person gets the same amount. (5 min.) • What models are used for multiplying/dividing fractions in the videos and Progressions? • What are the advantages to using different models of multiplying/dividing fractions?</td>
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<td>Slide 35</td>
<td>Multiplying/Dividing Fractions Discussion ?s</td>
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<td>Timing / Slide #s</td>
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<td><strong>15 min.</strong></td>
<td><strong>Slide 36</strong> Where are the Cookies? Questions for Discussion</td>
<td>📝</td>
<td>Hand out copy of <em>Where are the Cookies?</em> and direct individuals to read and work through the problem. (3 min.) Debrief using the following questions. 1. How would your students approach this problem? 2. What conceptual understanding of fractions does a student need to have in order to solve the previous problem? 3. What instructional strategies would you use to reach students at various levels of mathematical ability? (12 min.) *Try to steer participants toward different mathematical methods, not a focus on types of students (e.g. ELL, Special Ed).</td>
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<td><strong>Slide 37</strong></td>
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| **15 min.**      | **Slide 38** Questions for Further Investigation | 📝  | Small Group Discussion:  
• What opportunities should students be given to assist with building their conceptual understanding of fractions?  
• How do the various models of fractions build understanding? What are the consequences of a student being bound to one model (e.g. only using circles)? (7 min.) |
| **Slide 39**     | Questions for Further Investigation              |      | Whole Group Discussion:  
• How could various models have been used to facilitate understanding of any of the previous activities and what does the student’s choice of model tell the teacher about student |
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<tr>
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<th>Icon</th>
<th>Facilitator’s Information / Directions to Participants</th>
</tr>
</thead>
</table>
| 15 min. Slide 40 | Reflection for Further Discussion | ![People](image) | **Discuss as a whole group the following:** (3 separate clicks)  
- What type of FOCUS do I need in my grade level to help a student be successful on problems similar to those presented in today’s PD?  
- How do we work together within AND across grade levels to assure COHERENCE?  
- How do we maintain proper RIGOR in our instruction including: Conceptual Understanding, Fluency, and Application? |
| 2 min. Slide 41  | Next Steps | | Work with your students, gather student work, and re-visit and share students’ understanding and misconceptions with team or PLC.  
- What worked?  
- What didn’t?  
- Evaluate if individual students are ready to move on to next concept. |
| 2 min. Slide 42  | HW - Reflection of Conceptual Understanding (copy is at the end of guide) | ![Pencil](image) | ?s to reflect on:  
1. What opportunities should students be given to assist with building their conceptual understanding of fractions?  
2. How do the various models of fractions build understanding? List some possible outcomes of a student being bound to one model?  
3. How does your instruction allow for students to develop conceptual understanding of fractions? How is this embedded in your school’s math program?  
4. How do you provide opportunities for students to demonstrate conceptual understanding of fractions? How does your school’s math program support students’ demonstrating conceptual understanding of fractions? |
Three Shifts in Mathematics Instruction

There are three core shifts required by the Common Core State Standards (CCSS). By describing these three core shifts, we aim to ensure expectations for teaching and learning are clear, consistent, and tightly aligned to the goals of the Standards themselves.

**• Focus**

What type of **FOCUS** do I need in my grade level to help a student be successful on a problem such as this?

**• Coherence**

How do we work together within AND across grade levels to ensure **COHERENCE**?

**• Rigor**

How do we maintain proper **RIGOR** in our instruction including: Conceptual Understanding, Fluency, and Application?
Three key instructional shifts in the CCSS-M

1. **Focus**
   Strongly where the Standards Focus

   **Focus:** The Standards call for a greater focus in mathematics. Rather than racing to cover topics in today’s mile-wide, inch-deep curriculum, teachers use the power of the eraser and significantly narrow and deepen the way time and energy is spent in the math classroom. They focus deeply on the major work of each grade so that students can gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the math they have learned to solve problems inside and outside the math classroom.

2. **Coherence**
   Think across grades, and link to major topics within grades

   **Coherence:**
   Thinking across grades: The Standards are designed around coherent progressions from grade to grade. Principals and teachers carefully connect the learning across grades so that students can build new understanding onto foundations built in previous years. Teachers can begin to count on deep conceptual understanding of core content and build on it. Each standard is not a new event, but an extension of previous learning.

   Linking to major topics: Instead of allowing additional or supporting topics to detract from the focus of the grade, these topics can serve the grade level focus. For example, instead of data displays as an end in themselves, they support grade-level word problems.

3. **Rigor**
   in major topics pursue:
   • Conceptual understanding,
   • Procedural skill and fluency, and
   • Application with equal intensity.

   **Conceptual understanding:** The Standards call for conceptual understanding of key concepts, such as place value and ratios. Teachers support students’ ability to access concepts from a number of perspectives so that students are able to see math as more than a set of mnemonics or discrete procedures.

   **Procedural skill and fluency:** The Standards call for speed and accuracy in calculation. Teachers structure class time and/or homework time for students to practice core functions such as single-digit multiplication so that students have access to more complex concepts and procedures.

   **Application:** The Standards call for students to use math flexibly for applications. Teachers provide opportunities for students to apply math in context. Teachers in content areas outside of math, particularly science, ensure that students are using math to make meaning of and access content.

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Fraction Progressions Overview

In Grade 2, students build a basic understanding of fractions by partitioning a shape into 2, 3, or 4 equal-sized shares. Students describe these equal shares as halves, thirds, or fourths; they connect two halves, three thirds, or four fourths of a circle to model the relationship between the equal shares and the whole. While the term “fraction” may not necessarily be used at this time, this understanding prepares students for their formal study of fractions in Grade 3.

In Grade 3, students extend their work with geometric partitioning in grade 2 (2.G.3) to formally represent a unit fraction as $\frac{1}{b}$. Students relate $\frac{1}{2}$ to one of the two equal shares when a shape is partitioned into two equal parts. They also represent fractions on number lines by partitioning lines into line segments of equal length. This supports students’ understanding of fractions as numbers that can be represented as a unit fraction on the number line. By the end of Grade 3, students are able to construct a fraction from and decompose a fraction into its component unit fractions, e.g., $\frac{3}{4}$ decomposed into 3 equal segments of $\frac{1}{4}$ unit on the number line. *Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, & 8.

In Grade 4, students apply prior knowledge about composing and decomposing fractions, based on unit fractions, to adding and subtracting fractions with the same denominator, e.g., $\frac{3}{8} + \frac{4}{8} = \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right)$. They apply and extend previous understanding of multiplication with whole numbers and composing and decomposing fractions, based on unit fractions, to multiply a fraction by a whole number, e.g., $\frac{5}{8} = 5 \times \frac{1}{8}; \ 3 \times \frac{3}{4} = 9 \times \frac{1}{4}$ (4.NF.B.4a). They also learn and apply a fundamental property of equivalent fractions: multiplying the numerator and denominator of a fraction by the same whole number, generates an equivalent fraction. Students extend this understanding of the relative “size” of unit fractions and the composition of larger fractions from unit fractions to compare fractions with different numerators and different denominators (4.NF.A.2). Students build upon previous understanding of whole number place value by composing ten ones into tens, ten tens into hundreds, etc., to decomposing the standard unit into ten equal parts successively, e.g., 1 decomposed into ten one-tenths $(\frac{1}{10})$, one-tenth $(\frac{1}{10})$ decomposed into ten one-hundredths $(\frac{1}{100})$. This extends the Base
Ten number system beyond whole numbers into decimal representations of quantities. Students access their prior knowledge of adding ones and tens to understand the process for adding tenths and hundredths, e.g., by decomposing tenths into hundredths (4.NF.C). *Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, & 100.

**In Grade 5**, students apply their understanding of equivalent fractions and composing and decomposing fractions, based on unit fractions, to add and subtract fractions with unlike denominators, recognizing the need for a common denominator to complete the operation (5.NF.A). They interpret a fraction as division of the numerator by the denominator by applying their understanding from Grades 3 and 4 of division of whole numbers as partitioning into equal shares. Students apply prior understanding of partitioning shapes (Grade 2) to multiply fractions by fractions, e.g., partitioning $\frac{1}{2}$ of a shape into three (3) equal parts is the same as $\frac{1}{2} \times \frac{1}{3}$ (5.NF.B). Students use the relationship between multiplication and division to build understanding of the procedures for multiplying and dividing fractions.

**In Grades 6 and 7**, students use reasoning about multiplication and division to solve ratio and rate problems about quantities. Students begin to build an understanding of multiplication as scaling which reinforces previous work with multiplying fractions by whole numbers and fractions by fractions. Additional support for teaching ratio and proportional relationships can be found at [http://ime.math.arizona.edu/progressions/](http://ime.math.arizona.edu/progressions/)
FRACTION PROGRESSIONS

<table>
<thead>
<tr>
<th>GRADERS 2 &amp; 3</th>
<th>GRADE 4</th>
<th>GRADE 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>The meaning of fractions</td>
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<td>The number line and number line diagrams</td>
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<td>Equivalent Fractions</td>
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<td>Adding and subtracting fractions</td>
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<td>Comparing fractions</td>
<td>Comparing Fractions</td>
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<tr>
<td>Multiplication of a fraction by whole number</td>
<td>Multiplying and dividing fractions</td>
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<tr>
<td>Decimals</td>
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Equivalent Fractions

Using an area model show that:
\[
\frac{2}{3} = \frac{(4 \times 2)}{(4 \times 3)}
\]

4.NF.1 Explain why a fraction \( \frac{a}{b} \) is equivalent to a fraction \( \frac{(n \times a)}{(n \times b)} \) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Mrs. James left a tray of cookies on the counter early one morning. Larry walked by before lunch and decided to take 1/3 of the cookies on the tray. Later that afternoon Barry came in and ate 1/4 of the remaining cookies. After supper Terry saw the tray of cookies and ate 1/2 of the cookies remaining at that time. The next morning Mrs. James found the tray with only 6 cookies left. How many cookies were on the tray when Mrs. James first left it on the counter?

Which fraction is larger?

1. Using diagrams, determine which fraction in each pair is larger:
   a) \( \frac{4}{5}, \frac{4}{7} \)
   b) \( \frac{5}{6}, \frac{7}{8} \)
   c) \( \frac{3}{8}, \frac{2}{9} \)

2. What rules about the relative sizes of fractions can you state from these examples? Be as precise as you can in expressing your rules, without using the terms “numerator” or “denominator” or “top number” and “bottom number”.

3. What else can you find from the information given?
Reflection of Professional Learning w/ Additional Resources

1. What opportunities should students be given to assist with building their conceptual understanding of fractions?

2. How do the various models of fractions build understanding? List some possible outcomes of a student being bound to one model (e.g. only using circles)?

3. How does your instruction allow for students to develop conceptual understanding of fraction? How is this embedded in your school’s math program?

4. How do you provide opportunities for students to demonstrate conceptual understanding of fractions? How does your school’s math program support students’ demonstrating conceptual understanding of fractions?

After answering and reflecting on questions 1 – 4, use the resources in table 1 to further your understanding of fractions.
## Additional Resources Around Fractions

<table>
<thead>
<tr>
<th>Guide(s)</th>
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| **Developing Effective Fractions Instruction for Kindergarten Through 8th Grade**  
**NCEE 2010-4039 U.S. DEPARTMENT OF EDUCATION, September 2010**  
Other publications  

<table>
<thead>
<tr>
<th>Websites</th>
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| **Illuminations**  
http://illuminations.nctm.org/Reflections_3-5.html  
http://illuminations.nctm.org/reflections/3-5/Fractions/part1.html *(There are 3 parts to this.)*  
NCTM  
**Fraction Tracks (Game)** http://www.nctm.org/standards/content.aspx?id=26975  
**Part-Part Whole Game (Fractions)** math playground http://www.mathplayground.com/PartPartWhole.html  
**Mathlanding – Fraction tracks**  
http://www.mathlanding.org/content/fraction-tracks  
**Teaching Fractions with Understanding**  
http://nrich.maths.org/2550/index  
**Conceptual Math**  
http://www.conceptuamath.com/  
Supplemental online math curriculum for fractions and decimals – a few free resources, otherwise $ |

**NCTM Article:** *A Model for Understanding, Using, and Connecting Representations*  
Using a Journal Article as a Professional Learning Experience  
# Books

**Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 3-5 (Volume II)**  
by John Van de Walle, Karen S. Karp, Lou Ann H. Lovin, Jennifer M. Bay-Williams  

*Elementary and Middle Teaching Student-Centered Mathematics Series*  
*School Mathematics, Eighth Edition*, by John A. Van de Walle  

Companion website for *Elementary and Middle Teaching Student-Centered Mathematics Series*  
*SCHOOL MATHEMATICS* by John A. Van de Walle, Karen S. Karp, Jennifer M. Bay-Williams  

**Beyond Pizzas and Pies:**  
10 Essential Strategies for Supporting Fraction Sense  
by Julie McNamara and Meghan Shaughnessy  
Math Solutions Publications  
ISBN: 978-1-935099-13-0

**Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers**  
by Susan J. Lamon  
Routledge Publications  

**Teaching Arithmetic:**  
**Lessons for Introducing Fractions**  
by Marilyn Burns  
Math Solutions Publications  
ISBN: 978-0-941355-33-9

**Lessons for Extending Fractions**  
by Marilyn Burns  
Math Solutions Publications  
ISBN: 978-0-941355-43-8

**Teaching Arithmetic: Lessons for Multiplying and Dividing Fractions**  
by Marilyn Burns  
ISBN: 978-0-941355-64-3

**Zeroing in on Number and Operations, Grades 5–6**  
by Anne Collins and Linda Dacey  
Stenhouse Publishers  

**Zeroing in on Number and Operations, Grades 7–8**  
by Anne Collins and Linda Dacey  
Stenhouse Publishers  
Investigating Fractions, Decimals and Percents
by Catherine Twomey Fosnot and Colleagues
Heinemann Publishers
ISBN 978-0-325-01054-0

Extending Children's Mathematics:
Fractions and Decimals—Innovations in Cognitively Guided Instruction
by Susan Empson and Linda Levi
Heinemann Publishers
ISBN 978-0-325-03053-1