Facilitator's Guide – Fraction Progressions (CCSS-M)

WDE Contact: Laurie.Hernandez@wyo.gov

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Who

- Grade 2-6 Teachers in PLCs

Why

- To gain an understanding of the fraction progressions across a number of grade levels informed by research on children's cognitive development and the structure of mathematics.
- To build collaboration / articulation within and across grades.
- To further develop professional learning using additional resources by grade levels.

When

- In PLC's or building groups (not intended to be a 'one and done')
- Three - four hours with the possibility of breaking it up.

What (Content)

- Meaning and Modeling of fractions
- Comparing and equivalencies of fractions
- Operations with fractions

What for (Achievement-Based Objectives)

- To build content knowledge and understanding of the progressions through examination of a major topic across a number of grade levels informed by research on children's cognitive development and the structure of mathematics.
- To build collaboration/articulation within and across grades.

How (Learning Tasks)

- Begin with learning experience embedded in math…a math task on fractions that would lead to unpacking the progression document.
- Embed resources in the PD for use throughout the year.
- Pulling out salient points from the document, not reading the document.

The following structure does not include the following elements; you may want to include:

- Introduction
- Norms
- Breaks or Meals

Materials Needed

- Paper, pens/pencils, highlighters
- Chart Paper for small group share out
- Access to You Tube and Internet
- Speakers/System (connected to your computer so the videos can be heard by the group)

Copies (1/participant) - (printed and/or *electronic)

- Fraction Progressions Overview (copy at the end of this guide)
- Slide 3 – Shifts in Mathematical Instruction (copy at the end of this guide)
- Slide 7 – Fraction Progressions (copy at the end of this guide)
- Slide 14 (copy at the end of this guide)
- Cookie Activity (copy at the end of this guide)
- Reflection of Conceptual Understanding (copy at the end of this guide)
- *Progression Documents – http://ime.math.arizona.edu/progressions
- * Common Core State Standards for Mathematics (CCSS-M) – http://www.corestandards.org/Math
- Slide 7 - Blow up of Progressions Table – *optional* (can use chart paper, whiteboard, or smart board)

Icon Key

 **= Say to your audience**

 **= Show Video**

 **= Reading & Writing Activity**

 **= Writing Activity**

 **= Group Discussion (small or large)**

 **= Next Slide**

The following training contains 8 videos. You may decide, based on your audience, to show all of them or just a select few. Please feel free to adjust as needed.

Total Time: Approximately 4 hours

Possible Breakdown of Time for 1 hour bits:
Slides 1-12, 13-19, 20-31, 32-40 (time is estimated with all Videos & Activities)

Please consider doing a deep dive into this subject matter as it is a major shift for 3rd – 5th grade.

Timing / Slide #s	Title of Activity, Document, or Link to Webpage	Icon Direction	Facilitator's Information / Directions to Participants
5 min. **Slide 2** **Slide 3**	Agenda Objectives		• Objectives of Presentation • Fraction Overview • The Meaning of Fractions • Equivalent Fractions • Comparing Fractions • Operations with Fractions • Gain an understanding of the fraction progressions across grades 2-6, informed by research on children's cognitive development and the structure of mathematics. • Collaborate within and across grades. • Further develop professional learning using additional resources by grade level.
1 min. **Slide 4**	Key ?s for this Training (copy is at the end of guide)		*You'll need to perform 3 clicks. • What type of **FOCUS** do I need in my grade level to help a student be successful on a problem such as this? • How do we work together within AND across grade levels to ensure **COHERENCE?** • How do we maintain proper **RIGOR** in our instruction including: Conceptual Understanding, Fluency, and Application?
15 min. **Slide 5**	Fraction Overview (copy is at the end of guide)	 	Please refer to the Fraction Progressions Overview document. (15 min.) • Please read individually. • Underline the sentences which you think are the most important in unit development. • Share your sentences with the group. (Facilitator can decide if share out is with small groups or large group).

Timing / Slide #s	Title of Activity, Document, or Link to Webpage	Icon Direction	Facilitator's Information / Directions to Participants
5 min. **Slide 6**	The Overview of the Common Core Fractions Progressions Model http://youtu.be/X9NFEZlkoH0		Show video of the Overview. (3 min.)
15 min. **Slide 7-9**	Activity & Fraction Progressions (copy is at the end of guide)	   	Using your **CCSS-M Document**, work in pairs to identify the fraction standards in grades 2, 3, 4, or 5 and complete the Fraction Progressions Table. Complete chart in pairs by labeling the coding that fits. (i.e. 2.G.1 = grade.domain.cluster. The shaded boxes are to be left blank. Project Slide 7 and hand out copy of the ***Fraction Progressions.*** (5 min.) Share your findings with the group. (10 min.) Advance to Slide 8 to show filled in copy of chart.
2 min. **Slide 10**	Where are the Cookies?		Keep the following problem in mind as we work through today's activities. We will revisit it at the end of this training.
16 min. **Slide 11** **Slide 12**	Meaning of Unit Fractions http://vimeo.com/66775207#=0 Activity	 	Show video on Unit Fractions. (6 min.) • Read the section of the Progressions Document on development of the meaning of fractions and the number line.

Timing / Slide #s	Title of Activity, Document, or Link to Webpage	Icon Direction	Facilitator's Information / Directions to Participants
			• Work in pairs to answer the following question: (10 min.) • What are the important aspects of fractions that provide opportunities for mathematical practice of attending to precision?
10 min. **Slide 13**	Specifying the Whole Discussion	 	Read the gray boxes titled *The Importance of Specifying the Whole* AND *Area Representations of ¼.* • What is meant by "equal" parts?
15 min. **Slide 14** **Slides 15-16** **Slide 17**	Equivalent Fractions http://vimeo.com/68240897#at=0 Equivalent Fractions Activity (copy is at the end of guide) Discussion	   	Show video on equivalent fractions. (5 min.) Hand out copy ***Equivalent Fractions*** and have them work individually on the problem. (5 min.) Advance slide to show an example of a completed model. After watching the video and doing the activity, how has your perception of equivalent fractions and creating experiences for students about equivalent fractions changed? (5 min.)
7 min. **Slide 18**	Comparing Fractions http://vimeo.com/68253451#at=0		Show video on comparing fractions. (7 min.)

Timing / Slide #s	Title of Activity, Document, or Link to Webpage	Icon Direction	Facilitator's Information / Directions to Participants
20 min. **Slide 19**	Which Fraction is Larger?		Which fraction is larger? Note: The denominators in this problem are for the adults, not the students. Note: Depending on your group size, decide if each person does a, b, & c or if you break it out by groups. (5-10 min.)
Slide 20	Activity	 	• Read the section of the Progressions Document on Grade 4 Equivalent Fractions. • Work in pairs to answer the following question: • How can the use of area models and number line diagrams solidify a student's understanding of fraction comparison? (10 min.)
20 min. **Slide 21**	Adding Fractions http://vimeo.com/71842832#at=0		Show video on adding fractions. (6 min.)
Slide 22	Activity	 	Project slide and work through. (3 min.) [poss. use of chart paper] Discuss various approaches to this problem. Have participants display on chart paper.
Slides 23-24	Solutions		Advance slide (animated: 4 additional clicks) to see one possible approach and to next slide (animated: 3 additional clicks) to see the final solution. *Please point out that we have a unit (12/12 = 1) + a part of a unit.
Slide 25	Activity		• Read the section of the Progressions Document on Grade 4 and Grade 5 Adding and Subtracting Fractions.

Timing / Slide #s	Title of Activity, Document, or Link to Webpage	Icon Direction	Facilitator's Information / Directions to Participants

			• Work in pairs to answer the following question:
			  • How could a student build on their previous knowledge of adding/subtracting whole numbers in order to add/subtract fractions?
			  (5 min.)

15 min. **Slide 26** **Slide 27**	Multiplying Fractions (part 1) http://vimeo.com/71857774 Questions for Discussion	 	Show video on Multiplying Fractions (part 1). (4 min.) Work in pairs to, • Discuss how multiplying a fraction by a whole number is similar to/different from multiplying whole numbers. • Discuss some of the misconceptions students may have when multiplying a fraction by a whole number. (10 min.)
16 min. **Slide 28** **Slide 29**	Multiplying Fractions (part 2) http://vimeo.com/71859340#at=0 Questions for Discussion	 	Show video on Multiplying Fractions (part 2). (6 min.) *Optional stopping point to process and discuss.* (2:22) Work in pairs to, • Discuss one advantage and one disadvantage of using an area model when multiplying two fractions. • Create an area model that justifies each of your responses. (10 min.)

Timing / Slide #s	Title of Activity, Document, or Link to Webpage	Icon Direction	Facilitator's Information / Directions to Participants
8 min. **Slide 30** **Slide 31**	Questions for Discussion Possible Solution	 	With your partners, discuss how transparencies and color markers can be used to model the problem below: 3/4 x 2/5 = 3x2 / 4x5 (5 min.) Project slide for a demonstration of one possible solution.
10 min. **Slide 32**	Activity	 	• Read the section of the Progressions Document on Grade 4 and Grade 5 Multiplying and Dividing Fractions. (10 min.) • Work with a partner to respond to the following item: • Explain how creating a story/real-world context might assist a student in understanding fraction multiplication. (5 min.)
5 min. **Slide 33**	Dividing Fractions http://vimeo.com/71907763#at=0		Show video on Dividing Fractions. (5 min.)
15 min. **Slide 34** **Slide 35**	50 Pounds of Rice Multiplying/Dividing Fractions Discussion ?s	 	If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Using a visual model, show how each person gets the same amount. (5 min.) • What models are used for multiplying/dividing fractions in the videos and *Progressions*? • What are the advantages to using different models of multiplying/dividing fractions?

Timing / Slide #s	Title of Activity, Document, or Link to Webpage	Icon Direction	Facilitator's Information / Directions to Participants
			(10 min.)
15 min. **Slide 36** **Slide 37**	Where are the Cookies? Questions for Discussion	 	Hand out copy of ***Where are the Cookies?*** and direct individuals to read and work through the problem. (3 min.) Debrief using the following questions. 1. How would your students approach this problem? 2. What conceptual understanding of fractions does a student need to have in order to solve the previous problem? 3. What instructional strategies would you use to reach students at various levels of mathematical ability? (12 min.) *Try to steer participants toward different mathematical methods, not a focus on types of students (e.g. ELL, Special Ed).
15 min. **Slide 38** **Slide 39**	Questions for Further Investigation Questions for Further Investigation	 	Small Group Discussion: • What opportunities should students be given to assist with building their conceptual understanding of fractions? • How do the various models of fractions build understanding? What are the consequences of a student being bound to one model (e.g. only using circles)? (7 min.) Whole Group Discussion: • How could various models have been used to facilitate understanding of any of the previous activities and what does the student's choice of model tell the teacher about student

Timing / Slide #s	Title of Activity, Document, or Link to Webpage	Icon Direction	Facilitator's Information / Directions to Participants
			understanding? (7 min.)
15 min. **Slide 40**	Reflection for Further Discussion		**Discuss as a whole group the following:** (3 separate clicks) • What type of **FOCUS** do I need in my grade level to help a student be successful on problems similar to those presented in today's PD? • How do we work together within AND across grade levels to assure **COHERENCE?** • How do we maintain proper **RIGOR** in our instruction including: Conceptual Understanding, Fluency, and Application?
2 min. **Slide 41**	Next Steps		Work with your students, gather student work, and re-visit and share students' understanding and misconceptions with team or PLC. • What worked? • What didn't? • Evaluate if individual students are ready to move on to next concept.
2 min. **Slide 42**	HW - Reflection of Conceptual Understanding (copy is at the end of guide)		?s to reflect on: 1. What opportunities should students be given to assist with building their conceptual understanding of fractions? 2. How do the various models of fractions build understanding? List some possible outcomes of a student being bound to one model? 3. How does your instruction allow for students to develop conceptual understanding of fractions? How is this embedded in your school's math program? 4. How do you provide opportunities for students to demonstrate conceptual understanding of fractions? How does your school's math program support students' demonstrating conceptual understanding of fractions?

Slide 3

Three Shifts in Mathematics Instruction

There are three core shifts required by the Common Core State Standards (CCSS). By describing these three core shifts, we aim to ensure expectations for teaching and learning are clear, consistent, and tightly aligned to the goals of the Standards themselves.

•Focus

What type of **FOCUS** do I need in my grade level to help a student be successful on a problem such as this?

•Coherence

How do we work together within AND across grade levels to ensure **COHERENCE**?

•Rigor

How do we maintain proper **RIGOR** in our instruction including: Conceptual Understanding, Fluency, and Application?

Three key *instructional shifts* in the CCSS-M

1. **Focus** Strongly where the Standards Focus

Focus: The Standards call for a greater focus in mathematics. Rather than racing to cover topics in today's mile- wide, inch-deep curriculum, teachers use the power of the eraser and significantly narrow and deepen the way time and energy is spent in the math classroom. They focus deeply on the major work of each grade so that students can gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the math they have learned to solve problems inside and outside the math classroom.

2. **Coherence** think across grades, and link to major topics within grades

Coherence:
Thinking across grades: The Standards are designed around coherent progressions from grade to grade. Principals and teachers carefully connect the learning across grades so that students can build new understanding onto foundations built in previous years. Teachers can begin to count on deep conceptual understanding of core content and build on it. Each standard is not a new event, but an extension of previous learning.

Linking to major topics: Instead of allowing additional or supporting topics to detract from the focus of the grade, these topics can serve the grade level focus. For example, instead of data displays as an end in themselves, they support grade-level word problems.

3. **Rigor**: in major topics pursue:
   - Conceptual understanding,
   - Procedural skill and fluency, and
   - Application with equal intensity.

Conceptual understanding: The Standards call for conceptual understanding of key concepts, such as place value and ratios. Teachers support students' ability to access concepts from a number of perspectives so that students are able to see math as more than a set of mnemonics or discrete procedures.

Procedural skill and fluency: The Standards call for speed and accuracy in calculation. Teachers structure class time and/or homework time for students to practice core functions such as single-digit multiplication so that students have access to more complex concepts and procedures.

Application: The Standards call for students to use math flexibly for applications. Teachers provide opportunities for students to apply math in context. Teachers in content areas outside of math, particularly science, ensure that students are using math to make meaning of and access content.

Slide 4

Fraction Progressions Overview

In Grade 2, students build a basic understanding of fractions by partitioning a shape into 2, 3, or 4 equal-sized shares. Students describe these equal shares as halves, thirds, or fourths; they connect two halves, three thirds, or four fourths of a circle to model the relationship between the equal shares and the whole. While the term "fraction" may not necessarily be used at this time, this understanding prepares students for their formal study of fractions in Grade 3.

In Grade 3, students extend their work with geometric partitioning in grade 2 (2.G.3) to formally represent a unit fraction as $\frac{1}{b}$. Students relate $\frac{1}{2}$ to one of the two equal shares when a shape is partitioned into two equal parts. They also represent fractions on number lines by partitioning lines into line segments of equal length. This supports students' understanding of fractions as numbers that can be represented as a unit fraction on the number line. By the end of Grade 3, students are able to construct a fraction from and decompose a fraction into its component unit fractions, e.g., $\frac{3}{4}$ decomposed into 3 equal segments of $\frac{1}{4}$ unit on the number line. *Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, & 8.

In Grade 4, students apply prior knowledge about composing and decomposing fractions, based on unit fractions, to adding and subtracting fractions with the same denominator, e.g., $\frac{3}{8}+\frac{4}{8}=\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)+(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8})$. They apply and extend previous understanding of multiplication with whole numbers and composing and decomposing fractions, based on unit fractions, to multiply a fraction by a whole number, e.g., $\frac{5}{8}=5\times\frac{1}{8}$; $3\times\frac{3}{4}=9\times\frac{1}{4}$ (4.NF.B.4a). They also learn and apply a fundamental property of equivalent fractions: multiplying the numerator and denominator of a fraction by the same whole number, generates an equivalent fraction. Students extend this understanding of the relative "size" of unit fractions and the composition of larger fractions from unit fractions to compare fractions with different numerators and different denominators (4.NF.A.2). Students build upon previous understanding of whole number place value by composing ten ones into tens, ten tens into hundreds, etc., to decomposing the standard unit into ten equal parts successively, e.g., 1 decomposed into ten one-tenths $\left(\frac{1}{10}\right)$, one-tenth $\left(\frac{1}{10}\right)$ decomposed into ten one-hundredths $\left(\frac{1}{100}\right)$. This extends the Base

Ten number system beyond whole numbers into decimal representations of quantities. Students access their prior knowledge of adding ones and tens to understand the process for adding tenths and hundredths, e.g., by decomposing tenths into hundredths (4.NF.C). *Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, & 100.

In Grade 5, students apply their understanding of equivalent fractions and composing and decomposing fractions, based on unit fractions, to add and subtract fractions with unlike denominators, recognizing the need for a common denominator to complete the operation (5.NF.A). They interpret a fraction as division of the numerator by the denominator by applying their understanding from Grades 3 and 4 of division of whole numbers as partitioning into equal shares. Students apply prior understanding of partitioning shapes (Grade 2) to multiply fractions by fractions, e.g., partitioning $\frac{1}{2}$ of a shape into three (3) equal parts is the same as $\frac{1}{2} \times \frac{1}{3}$ (5.NF.B). Students use the relationship between multiplication and division to build understanding of the procedures for multiplying and dividing fractions.

In Grades 6 and 7, students use reasoning about multiplication and division to solve ratio and rate problems about quantities. Students begin to build an understanding of multiplication as scaling which reinforces previous work with multiplying fractions by whole numbers and fractions by fractions. Additional support for teaching ratio and proportional relationships can be found at http://ime.math.arizona.edu/progressions/

Slide 7

FRACTION PROGRESSIONS

GRADES 2 & 3	**GRADE 4**	**GRADE 5**
The meaning of fractions		
The number line and number line diagrams		
Equivalent Fractions	Equivalent Fractions	
	Adding and subtracting fractions	Adding and subtracting fractions
Comparing fractions	Comparing Fractions	
	Multiplication of a fraction by whole number	Multiplying and dividing fractions
		Multiplication as scaling
	Decimals	

Slide 14

Equivalent Fractions

Using an area model show that:

$$\frac{2}{3} = \frac{(4 \times 2)}{(4 \times 3)}$$

4.NF.1 Explain why a fraction $\frac{a}{b}$ *is equivalent to a fraction* $\frac{(n \times a)}{(n \times b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Slide 35

Where are the cookies?

Mrs. James left a tray of cookies on the counter early one morning. Larry walked by before lunch and decided to take 1/3 of the cookies on the tray. Later that afternoon Barry came in and ate 1/4 of the remaining cookies. After supper Terry saw the tray of cookies and ate 1/2 of the cookies remaining at that time. The next morning Mrs. James found the tray with only 6 cookies left. How many cookies were on the tray when Mrs. James first left it on the counter?

Which fraction is larger?

1. Using diagrams, determine which fraction in each pair is larger:
   a) $\frac{4}{5}, \frac{4}{7}$
   b) $\frac{5}{6}, \frac{7}{8}$
   c) $\frac{3}{8}, \frac{2}{9}$

2. What rules about the relative sizes of fractions can you state from these examples? Be as precise as you can in expressing your rules, without using the terms “numerator” or “denominator” or “top number” and “bottom number”.

3. What else can you find from the information given?

Reflection of Professional Learning w/ Additional Resources

1. What opportunities should students be given to assist with building their conceptual understanding of fractions?

2. How do the various models of fractions build understanding? List some possible outcomes of a student being bound to one model (e.g. only using circles)?

3. How does your instruction allow for students to **develop** conceptual understanding of fraction? How is this embedded in your school's math program?

4. How do you provide opportunities for students to **demonstrate** conceptual understanding of fractions? How does your school's math program support students' demonstrating conceptual understanding of fractions?

After answering and reflecting on questions 1 – 4, use the resources in table 1 to further your understanding of fractions.

Additional Resources Around Fractions

Guide(s)

Developing Effective Fractions Instruction for Kindergarten Through 8th Grade
http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=15

NCEE 2010-4039 U.S. DEPARTMENT OF EDUCATION, September 2010

Other publications
http://ies.ed.gov/ncee/wwc/publications_reviews.aspx

Websites

Illuminations
http://illuminations.nctm.org/Reflections_3-5.html
http://illuminations.nctm.org/reflections/3-5/Fractions/part1.html (There are 3 parts to this.)
NCTM
Fraction Tracks (Game) http://www.nctm.org/standards/content.aspx?id=26975

Part-Part Whole Game (Fractions) **math playground** **http://www.mathplayground.com/PartPartWhole.html**

Mathlanding – Fraction tracks
http://www.mathlanding.org/content/fraction-tracks

Teaching Fractions with Understanding
http://nrich.maths.org/2550/index

Conceptual Math
http://www.conceptuamath.com/
Supplemental online math curriculum for fractions and decimals – a few free resources, otherwise $

NCTM Article: *A Model for Understanding, Using, and Connecting Representations*
Using a Journal Article as a Professional
Learning Experience
http://www.nctm.org/publications/article.aspx?id=21548

Books

Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 3-5 (Volume II)
by John Van de Walle, Karen S. Karp, Lou Ann H. Lovin, Jennifer M. Bay-Williams Pearson Publications: ISBN-13: 978-0132824873 | Edition: 2

Elementary and Middle **Teaching Student-Centered Mathematics Series** *School Mathematics, Eighth Edition,* by John A. Van de Walle Pearson Publications: ISBN-13: 978-0132900973 | Edition: 8

Companion website for *Elementary and Middle* **Teaching Student-Centered Mathematics Series** *School Mathematics* by John A. Van de Walle, Karen S. Karp, Jennifer M. Bay-Williams
http://wps.ablongman.com/ab_vandewalle_math_6/0%2C12312%2C3547873-%2C00.html

Beyond Pizzas and Pies:
10 Essential Strategies for Supporting Fraction Sense
by Julie NcNamara and Meghan Shaughnessy
Math Solutions Publications
ISBN: 978-1-935099-13-0

Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers
by Susan J. Lamon
Routledge Publications
ISBN-13: 978-0415886123 | Edition: 3

Teaching Arithmetic:
Lessons for Introducing Fractions
by Marilyn Burns
Math Solutions Publications
ISBN: 978-0-941355-33-9

Lessons for Extending Fractions
by Marilyn Burns
Math Solutions Publications
ISBN: 978-0-941355-43-8

Teaching Arithmetic: Lessons for Multiplying and Dividing Fractions
by Marilyn Burns
ISBN: 978-0-941355-64-3

Zeroing in on Number and Operations, Grades 5–6
by Anne Collins and Linda Dacey
Stenhouse Publishers
ISBN: 978-157110-798-5

Zeroing in on Number and Operations, Grades 7–8
by Anne Collins and Linda Dacey
Stenhouse Publishers
ISBN: 978-157110-799-2

Investigating Fractions, Decimals and Percents
by Catherine Twomey Fosnot and Colleagues
Heinemann Publishers
ISBN 978-0-325-01054-0

Extending Children's Mathematics:
Fractions and Decimals—Innovations in Cognitively Guided Instruction
by Susan Empson and Linda Levi
Heinemann Publishers
ISBN 978-0-325-03053-1